MATHEMATICS

2021 - 2022

غير مصرح بتداول هذا الكتاب خارج
وزارة التربية والتعليم والتعليم الفنى

For Preparatory Year Two

First Term

Student's Book

Authors

Mr. Omar Fouad Gaballa

Prof.Dr. Afaf Abo-ElFoutoh Salah

Dr. Essam Wasfi Rouphaeil

Mr. Mahmoud Yasser Al-Khatieb

Mr. Serafiem Elias Iskander

Revised by Samir Mohamed Sedawy

Counsellor of Mathematics

M. O. E

Mr. Gamal El shahed

بسم الله الرحمن الرحيم

Dear students:

It is extremely great pleasure to introduce the mathematics book for second preparatory. We have been specially cautious to make your learning to the mathematics enjoyable and useful since it has many practical applications in real life as well as in the other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate the mathematicians roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining the patterns of positive thinking which pave your way to creativity .

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration .

Our great interest here is to help you get the information by your self in order to develop your self-study skills.

Calculators and computer sets are used when there's a need for. Exercises, practices, general exams, portfolios, unit test, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

Authors

Contents

Unit One : Real Numbers

Unit Two : Relation between two Variables

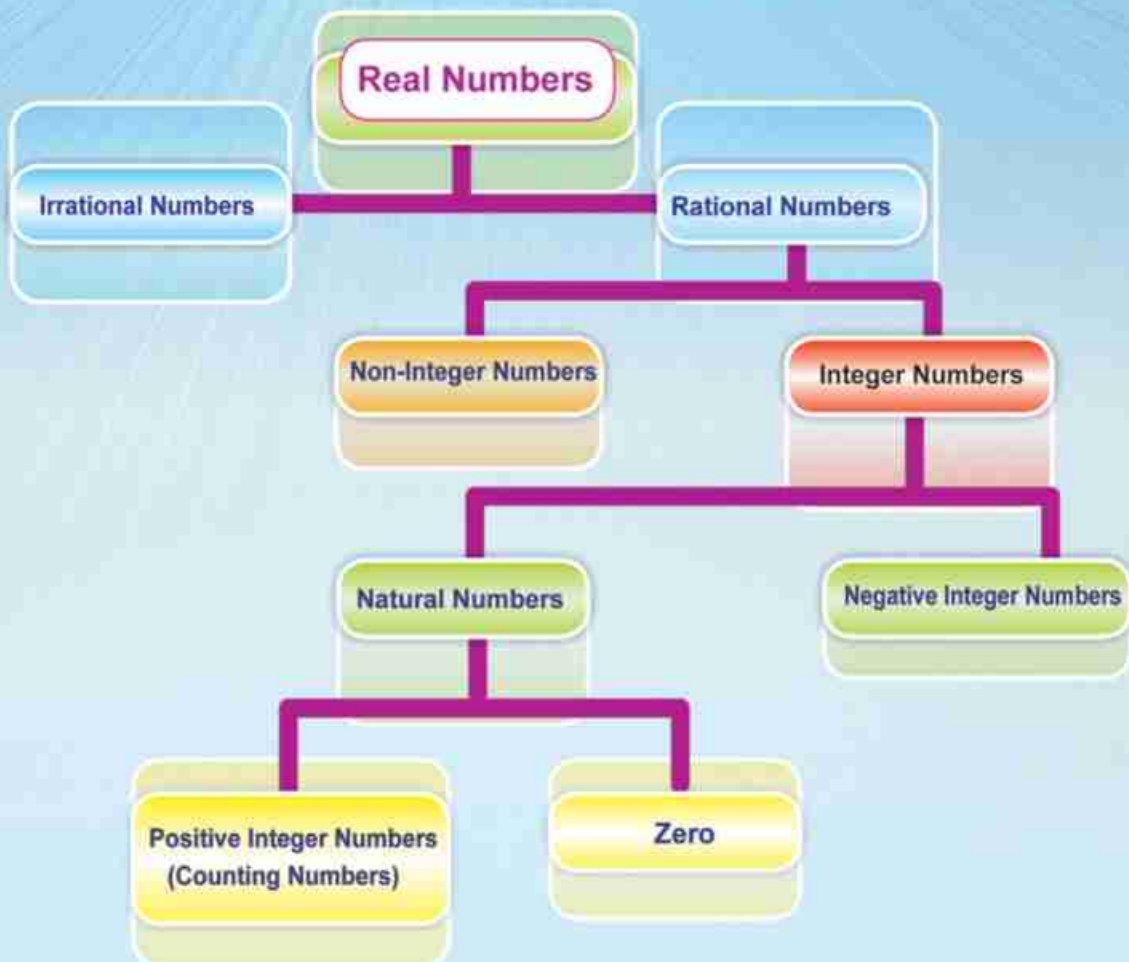
Unit Three : Statistics

Unit Four: Geometry

Unit Five : Inequality

The used Mathematical Symbols

$\mathbb{N}$	The set of natural numbers	$\perp$	perpendicular to
$\mathbb{Z}$	The set of integer numbers	$//$	parallel to
$\mathbb{Q}$	The set of rational numbers	$\overline{AB}$	Line segment AB
$\mathbb{Q}'$	The set of irrational numbers	$\overrightarrow{AB}$	Ray AB
$\mathbb{R}$	The set of real numbers	$\overleftrightarrow{AB}$	straight line AB
$\sqrt{a}$	Square root of number a	$m(\angle L)$	measure of angle L
$\sqrt[3]{a}$	Cube root of number a	$\sim$	Similarity
$[a, b]$	Closed interval	$<$	less than
$]a, b[$	Open interval	$\leq$	less than or equal to
$[a, b[$	Half-open (closed) interval	$>$	greater than
$]a, b]$	Half-open (closed) interval	$\geq$	greater than or equal to
$]-\infty, a]$ $[a, \infty[$	Infinite interval	$P(E)$	probability of occurring event (E)
$\equiv$	is congruent to		

UNIT ONE
1
Real Numbers
Real Numbers
Irrational Numbers
Rational Numbers
Non-Integer Numbers
Integer Numbers
Natural Numbers
Negative Integer Numbers
Positive Integer Numbers
(Counting Numbers)
Zero

Revision

Think and Discuss

The sets of numbers

The set of Counting numbers = {1, 2, 3, ...}

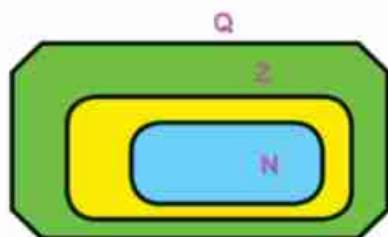
The set of Natural numbers : $\mathbf{N}$ = {0, 1, 2, 3, ...} = counting numbers $\cup$ {0}

The set of Integers : $\mathbf{Z}$ = { ..., -3, -2, -1, 0, 1, 2, 3,...}

The set of Positive integers $\mathbf{Z}^+$ = { 1, 2, 3, ...} = Counting numbers

The set of Negative integers $\mathbf{Z}^-$ = { -1, -2, -3, ...}

$$Z = Z^+ \cup \{0\} \cup Z^-$$

The set of Rational numbers Q = $\{\frac{a}{b} : a, b \in Z, b \neq 0\}$

$$N \subset Z \subset Q$$

The absolute value of a rational number:

$|-7| = 7$, $|3| = 3$, $|0| = 0$, $|-\frac{5}{3}| = \frac{5}{3}$

If $|a| = 5$ then $a = \pm 5$

The Standard form of a rational number is :

$a \times 10^n$ where $n \in z$, $1 \leq | a | < 10$

For example:- The standard form of the number 25.32×10^4
$= 2.532 \times 10^5$

- The standard form of the number $0.00053 = 5.3 \times 10^{-4}$

The perfect square rational number:

It is that positive number which can be written in the form of a square rational number i.e (rational number)^2

Example 1 , 4 , 25 , $\frac{9}{16}$, $2\frac{1}{4}$, ...

The perfect cube of rational number:

It is that rational number which can be written in the form of a cube rational number. i.e (rational number)^3

Example 1 , 8 , -27 , -216 , $\frac{8}{125}$, ...

The square root of a perfect square rational number

- The square root of the positive rational number a is that number whose square is equal to a.
- ($\sqrt{zero}$ = zero) the square root of zero is zero.
- Every perfect square rational number a has two square roots each one of them is an additive inverse to the other i.e. $\sqrt{a}$, $-\sqrt{a}$

Example $\frac{16}{25}$ has two square roots: $\frac{4}{5}$, $-\frac{4}{5}$

- $\sqrt{9}$ means the positive square root of 9 which is equal to 3
- $\sqrt{\left(\frac{a}{b}\right)^2} = \left|\frac{a}{b}\right|$ i.e
  $\sqrt{(-7)^2} = |-7| = 7$

Practice

Complete the following table

Number	Natural Number	Integer	Rational Number
3	✓	✓	✓
-3			
$\frac{3}{5}$			
$\sqrt{\frac{9}{16}}$			
$\|5 - 7\|$			

Revision exercises

1 **Complete** **Write the following numbers in the form $\frac{a}{b}$ where a and b are two integers and there are n`t common factors between them, b ≠ zero.**

A. $0,2 =$ ....
B. $0,3 =$ ....
C. $25\% =$ ....
D. $|-0,75| =$ ....
E. $-6 =$ ....
F. $1\frac{1}{4} =$ ....

2 **Choose** **the correct answer:**

A. The solution set of the equation $x + 5 = |-5|$ in N is .... ({0} or {10} or {-10}, Ø)
B. The rational number lies between $\frac{1}{5}$, $\frac{2}{5}$ is ....($\frac{2}{10}$ or $\frac{1}{10}$ or 0.3 or -0.3)
C. The product of the rational number $\frac{a}{b}$ by its additive inverse is ..... (zero, $-\frac{a}{b}$ or $\frac{a^2}{b^2}$ or $\frac{-a^2}{b^2}$)
D. $|-2| + |-4| + |6| =$ .... (zero, $|-12|$, -12 , 6)
F. $\sqrt{a^2} =$ .... (a or -a or $|a|$ or $\pm a$)

3 **Find** **the value of x which satisfies each of the following equations. Determine whether this value is a natural, real or rational number.**

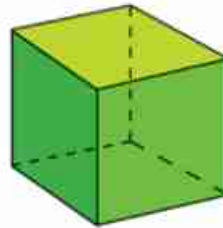
A. $5X + 3 = 20$
B. $7X + 11 = 12$
C. $3X + 5 = 1$
D. $X + 3 = 7$

4 **Find** **in the simplest form:**

A. $\sqrt{25 + 144} =$ ....
B. The standard form of 0 00015 is ....
C. $\sqrt{0.16} + |-0.6| =$ ....
D. $2^0 + 2^1 + 2^2 + 2^3 =$ ....
E. The sum of the two square roots of the number $2\frac{1}{4} =$ ....
F. $\sqrt{0.25} =$ ....

The cube root of a rational number

Think and Discuss

you have learned that:
The volume of a cube =
the length of its side × itself × itself

Complete

The volume of the cube whose side length is equal to 7 cm
= .... × .... × .... = ... cm^3

Let's think

125	5
25	5
5	5
1	

If we have a cube of volume 125 cm^3, what is the length of its side?
We search for any three equal numbers of a product equal to 125. Then the number 125 can be factorized into its prime factors
$125 = 5 \times 5 \times 5$

$\therefore$ the cube of volume 125cm^3 has a side length = 5cm
Therefore, 5 is called the cube root of 125 and it is written as $\sqrt[3]{125} = 5$.

The cube root of the rational number a is that number whose cube is equal to a

- The cube root for the rational number a is symbolized by $\sqrt[3]{a}$
- The cube root for a positive rational number is also positive
  Ex: $\sqrt[3]{125} = 5$
- - The cube root for a negative rational number is also negative. Ex: $\sqrt[3]{-8} = -2$ why ?
- $\sqrt[3]{zero}$ = zero
- $\sqrt[3]{a^3} = a$

you will learn how

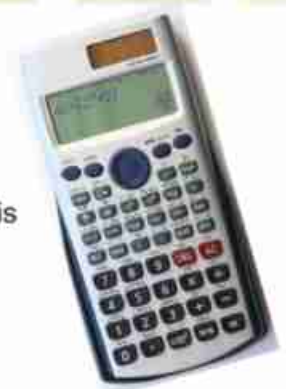
- To find the cube root of a rational number using facorization.
- To find the cube root of a rational number using the calculator.
- To solve equations that include finding the cube root.
- To solve applications on the cube root of a rational number.

Key terms

- Cube root .

To find the cube root of a perfect cube rational number:

- The number can be factorized into its prime factors..
- A calculator can be used.

Remark The perfect cube rational number has one cube root which is also a rational number , why?

Examples

1 **Use factorization to find the value of each $\sqrt[3]{1000}$, $\sqrt[3]{-216}$, $\sqrt[3]{3\frac{3}{8}}$; then check your answer using the calculator.**

Solution

	2	1000
2	2	500
	2	250
	5	125
5	5	25
	5	5
		1

$\sqrt[3]{1000} = 5 \times 2 = 10$

	2	216
2	2	108
	2	54
	3	27
3	3	9
	3	3
		1

$\sqrt[3]{-216} = -2 \times 3 = -6$

$3\frac{3}{8} = \frac{27}{8}$

3	27	2	8
3	9	2	4
3	3	2	2
	1		1

$\sqrt[3]{3\frac{3}{8}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

Use your calculator to check your answer by pressing on $\sqrt[3]{\square}$

2 **Find the length of the radius of a sphere whose volume is equal to 4851cm³ ($\pi = \frac{22}{7}$)**

Solution

The volume of the sphere $= \frac{4}{3}\pi r^3$

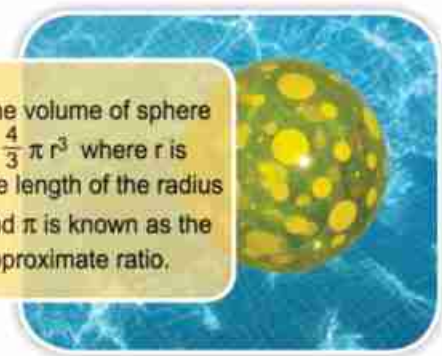
$4851 = \frac{4}{3} \times \frac{22}{7} r^3$

$r^3 = \frac{4851 \times 3 \times 7}{4 \times 22} = \frac{9261}{8}$

$\therefore r^3 = \frac{3^3 \times 7^3}{2^3}$

$\therefore r = \sqrt[3]{\frac{3^3 \times 7^3}{2^3}}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

The volume of sphere $= \frac{4}{3}\pi r^3$ where r is the length of the radius and π is known as the approximate ratio.

$r = \frac{3 \times 7}{2} = \frac{21}{2} = 10.5$ cm

we can use the calculator to find $\sqrt[3]{\frac{9261}{8}}$ directly.

Find the diameter of the sphere whose volume is 113.04 cm^3 ($\pi = 3.14$)

Example

Solve each of the following equations in Q.

A $x^3 = 8$

B $x^3 + 9 = 8$

C $(x - 2)^3 = 125$

D $(2x - 1)^3 - 10 = 54$

Solution

A $x^3 = 8$

$x = \sqrt[3]{8} = 2$

$\therefore$ Solution set= {2}

B $x^3 + 9 = 8$

$x^3 = 8 - 9$

$x^3 = -1$

$x = \sqrt[3]{-1} = -1$ $\therefore$ Solution set = {-1}

C $(x - 2)^3 = 125$

$x - 2 = \sqrt[3]{125}$

$x - 2 = 5$

$x = 7$

$\therefore$ Solution set = {7}

D $(2x - 1)^3 - 10 = 54$

$(2x - 1)^3 = 64$

$2x - 1 = \sqrt[3]{64}$

$2x - 1 = 4$

$2x = 5$

$x = \frac{5}{2}$ $\therefore$ Solution set= { $\frac{5}{2}$ }

Solve the following equations in Q: $(x + 1)^3 = 27$, $(x + 1)^3 = -27$

Exercises (1-1)

1 **Fill in the following table:**

Number a	8	125	-27	....	$3\frac{3}{8}$	$-\frac{8}{125}$	....	....
$\sqrt[3]{a}$	....	....	....	-10	....	....	6	-4

2 **Complete**

A) $\sqrt[3]{-125}$ = ... B) $\sqrt[3]{343}$ = ... C) $\sqrt[3]{8} + \sqrt[3]{-8}$ = ...

D) $\sqrt[3]{0.001}$ = ... E) $\sqrt[3]{27} - \sqrt[3]{64}$ = ... F) $\sqrt[3]{a^3}$ = ...

3 **Choose the correct answer from the Parentheses:**

A) $\sqrt[3]{(-8)^2}$ = ... (2 or -2 or 4 or -4)

B) $\sqrt{25} - \sqrt[3]{-125}$ = ... (10 or 0 or 5 or ± 5)

C) $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25}$ = ... ($\frac{3}{2}$ or $\frac{1}{2}$ or 2 or -2)

D) $\sqrt[3]{1000} \times \sqrt[3]{-0.008}$ = ... ($\frac{1}{2}$ or 10 or 2 or -2)

E) The lateral area of a cube whose volume is 216 cm^3 = ...cm^2 (36 or 6 or 144 or 216)

F) $\sqrt[3]{x^6} = \sqrt{\ldots\ldots}$ (x^3 or x^2 or x or x^4)

E) $\sqrt[3]{-27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0.125}$ = ... (1 or 0 or -1 or $\frac{11}{2}$)

4 **Find the value of x in each of following cases:**

A) $\sqrt[3]{x} = 5$ B) $\sqrt[3]{x} = -\frac{1}{2}$ C) $\sqrt[3]{x} = -\sqrt{4}$

D) $x^3 = -8$ E) $x^3 - 125 = 0$ F) $x^2 = 64$

5 **Find the solution set for each of the following equations in Q:**

A) $x^3 + 27 = 0$ B) $8x^3 + 7 = 8$

C) $(x + 3)^3 = 343$ D) $(5x - 2)^3 + 10 = 18$

6 **Application problems**

A) Find the side length of a cube vessel with capacity of one liter.

B) Find the diameter length of a sphere whose volume = $\frac{1372}{81}\pi$ cubic unit (Use $\pi = \frac{22}{7}$) (volume of a sphere = $\frac{4}{3}\pi r^3$)

The set of Irrational numbers Q`

Think and Discuss

you will learn how how

- To define the set of irrational numbers.

key terms

- Irrational number

you have learned that: A rational number is that number which can be put in the form:

$$\frac{a}{b} : \text{where } a , b \in z,\ b \neq 0$$

for example: when solving the equation $4x^2 = 25$

then $x^2 = \frac{25}{4}$ $\therefore x = \pm \frac{5}{2}$

Remark Each of $\frac{5}{2}$, $-\frac{5}{2}$ is a rational number .

However, there are many numbers which can not be put in the form $\frac{a}{b}$ where $a , b \in Z , b \neq 0$

for example : when solving the equation $X^2 = 2$, we can not find any rational number whose square is equal to 2

The irrational number It is that number which can not be put in the form $\frac{a}{b}$ where $a , b \in \mathbb{Z}, b \neq 0$

the following are examples to irrational numbers.

First : the square roots of the positive numbers which are not perfect squares

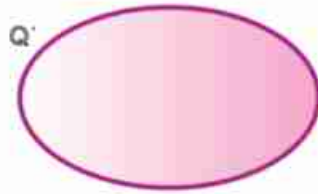
Ex : $\sqrt{2}$, $\sqrt{5}$, $-\sqrt{6}$, $\sqrt{7}$

Second: the cube roots of those numbers that are not perfect cubes

Ex : $\sqrt[3]{4}$, $\sqrt[3]{-2}$, $\sqrt[3]{11}$, ...

Third: the pi π (the approximation ratio)

Where it is impossible to find any exact value for any of the previous number. why?

Those numbers and others form a set which is called the set of irrational numbers which is denoted by the symbol Q`.

Think : is $\sqrt[3]{-1}$ an irrational number? why?

Exercises (1-2)

1 **Complete** :using the symbol Q or Q`.

A) $5 \in$ .... B) $\sqrt[3]{10} \in$ .... C) $0 \in$ ....

D) $-0.7 \in$ .... E) $\sqrt[3]{8} \in$ .... F) $\sqrt[3]{6} \in$ ....

G) $\sqrt[3]{-9} \in$ .... H) $\pi \in$ ....

2 **Put (✔) on the true sentence and (✕) on the false sentence**

A) $2.3 \times 10^5 \in Q$ ( ) B) $|-5| \in Q`$ ( ) C) $\frac{zero}{5} \in Q$ ( )

D) $\sqrt[3]{-4} \in Q`$ ( ) E) $\sqrt{1000} \in Q$ ( ) F) $\sqrt{7} > 3$ ( )

G) $\sqrt[3]{10} > 2$ ( ) H) $\sqrt[3]{20} > \sqrt{9}$ ( )

I) The side length of a square whose surface area = $6cm^2$ is a rational number ( )

3 **Choose the correct answer:**

A) The surface area of a square whose side length is $\sqrt{3}$ = ...

($4\sqrt{3}$ or 9 or 3 or 6)

B) The irrational number lies between 3 and 4 is ....

(3.5 or $\frac{1}{8}$ or $\sqrt{13}$ or $\sqrt{20}$)

C) The irrational number lies between -2 and -1 is ....

(-3 or $-1\frac{1}{2}$ or $-\sqrt{3}$ or $\sqrt{2}$)

Finding the approximate value of an Irrational number

Think and discuss

Can you find the two rational numbers which the irrational number $\sqrt{2}$ is located between them.

Remark $\sqrt{2}$ is between $\sqrt{1}$, $\sqrt{4}$ i.e $1 < \sqrt{2} < 2$

i.e. $\sqrt{2} = 1 +$ a decimal fraction

To find the approximate value of $\sqrt{2}$. We check the values of the following numbers:

$(1.1)^2 = 1.21$, $(1.2)^2 = 1.44$, $(1.3)^2 = 1.69$,

$(1.4)^2 = 1.96$, $(1.5)^2 = 2.25$

$\because 1.96 < 2 < 2.25$

$\therefore 1.4 < \sqrt{2} < 1.5$

i.e. $\sqrt{2} = 1.4 +$ a decimal fraction

i.e. $1.41 < \sqrt{2} < 1.42$

Use the calculator to check you answer.

You will learn how

- To find the approximate value for an irrational number
- To represent an irrational number on the number line.
- To solve equations in Q`

Representing the irrational number on the number line.

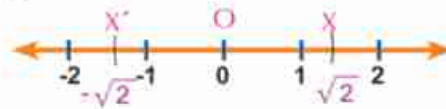
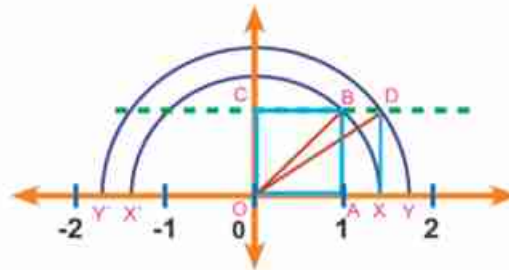
How can the point represents $\sqrt{2}$ be located on the number line?

If we draw the right triangle ABC at B which is an isosceles triangle also.

where AB = BC = one unit of length

Then $(AC)^2 = (AB)^2 + (BC)^2 = 1^2 + 1^2 = 2$

$\therefore AC = \sqrt{2}$ unit of length.

- draw the number line and place the sharp point of the compasses at point O, then adjust the compasses to a length that is equal to $\overline{AC}$ and draw an arc that intersects the number line on the right of o and at the point X, where that point represents $\sqrt{2}$
- Using the same length, we can label the point X` which represent $-\sqrt{2}$ where X` is on the left of the point o.

The length of its diagonal = $\sqrt{1+1} = \sqrt{2}$ unit of length

$\therefore$ OB = $\sqrt{2}$

- Place the sharp point of the compasses at point O and draw a semi-circle whose diameter = the length of $\overline{OB} = \sqrt{2}$.
- $\overleftrightarrow{OA} \cap$ the semi-circle = {X , X`} where X represents the number $\sqrt{2}$, x` represents the number $-\sqrt{2}$.
- Draw $\overline{XD} \parallel \overline{AB}$ and intersects $\overleftrightarrow{CB}$ at D

  $(OD)^2 = (OX)^2 + (XD)^2 = (\sqrt{2})^2 + (1)^2 = 3$

  $\therefore$ OD = $\sqrt{3}$
- Place the sharp point of the compasses at point O and adjust it to a length which is equal to the length of $\overline{OD}$, then draw semi-circle that intersects with $\overleftrightarrow{OA}$ at points Y, Y`

$\therefore$ OY = $\sqrt{3}$ **i.e.** point Y represnts $\sqrt{3}$, while point Y` represents $-\sqrt{3}$

- Continue using the same method to represent $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, ...

  also $-\sqrt{4}$, $-\sqrt{5}$, $-\sqrt{6}$, ...

Practice

1 **Find :**

- A. Two consecutive integers that $\sqrt{5}$ lies between them.
- B. Two consecutive integers that $\sqrt{12}$ lies between them.
- C. Two consecutive integers that $\sqrt[3]{10}$ lies between them.
- D. Two consecutive integers that $\sqrt[3]{-20}$ lies between them.

2 **Prove that :**

- A. $\sqrt{3}$ lies between 1.7 , 1.8 .
- B. $\sqrt[3]{15}$ lies between 2.4 , 2.5 .

3 Find the value of $\sqrt{11}$ to the nearest hundredth.

4 Find the value of $\sqrt[3]{2}$ to the nearest tenth.

5 Draw the number line and label the point which represents the irrational number $\sqrt{3}$.

6 Draw the number line and label the point which represents the irrational number $1+\sqrt{2}$

Example (1)

Find the solutions set for each of the following equations in Q`:

A. $x^2 = 2$ B. $x^3 = 5$ C. $\frac{4}{3}x^2 = 1$ D. $0.001\,x^3 = -8$

Solution

A. $x^2 = 2$

$\therefore x = \pm\sqrt{2}$ Solution set = $\{-\sqrt{2}\ ,\ \sqrt{2}\}$

B. $x^3 = 5$

$\therefore x = \sqrt[3]{5}$ Solution set = $\{\sqrt[3]{5}\}$

C. $\frac{4}{3}x^2 = 1$

$\therefore \frac{3}{4} \times \frac{4}{3}x^2 = \frac{3}{4} \times 1$

$x^2 = \frac{3}{4}$

$\therefore x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$ Solution set $= \{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$

D $0.001\ x^3 = -8$

$$x^3 = -\frac{8}{0.001} = -8000$$

$$\therefore x = \sqrt[3]{-8000}$$

$$= -20 \in Q$$

The solution set in $Q' = \varnothing$

Example (2)

Find the length of each of the side and the diagonal of a square whose area is $7cm^2$.

Solution

Let the length of the side be x cm,

then the area = $x \times x = x^2$

Where L is the square diagonal length

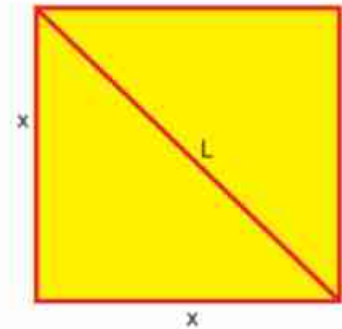
$x^2 = 7$

$\therefore x = \pm\sqrt{7}$ cm $\qquad \therefore x = \sqrt{7}$ cm why?

To find the diagonal of the square: use pythagorean theorem

$L^2 = x^2 + x^2$ Where L is the square diagonal length.

$\therefore L^2 = 14$

$\therefore L = \pm\sqrt{14}$ cm $\qquad \therefore L = \sqrt{14}$ cm why?

Example (3)

Find : the circumference of a circle whose area is 3π cm²

Solution

The area of the circle = πr^2

$$3\pi = \pi r^2$$

$$\therefore r^2 = 3$$

$r = \sqrt{3}$ cm $\qquad$ or $r = -\sqrt{3}$ cm (refused)

the circumference = $2\pi r = 2\pi \times \sqrt{3} = 2\sqrt{3}\,\pi$ cm.

Exercises (1-3)

1 Circle the irrational number in each of the following:

$\sqrt{3}$, -0,2 , $\sqrt[3]{-1}$, 0 , $\sqrt[3]{9}$, $-\sqrt{\frac{4}{25}}$

2 **Find** the value of X in each of the following cases and determine whether

$x \in Q$ or $\mathbf{x} \in Q'$

A) $4x^2 = 9$ B) $2x^2 = 6$ C) $x^3 = 125$

D) $x^3 = 10$ E) $(x - 1)^2 = 4$ F) $(x - 2)^3 = 1$

3 **Find** the approximate value for $\sqrt{10}$, and check your answer using the calculator.

4 **Think** : If x is an integer, find the value of X in each the following cases:

A) $x < \sqrt{7} < x + 1$ B) $x < \sqrt{80} < x + 1$ C) $x < \sqrt{125} < x + 1$

D) $x < \sqrt[3]{5} < x + 1$ E) $x < \sqrt[3]{30} < x + 1$ F) $x < \sqrt[3]{100} < x + 1$

5 **Choose** the correct answer :

A) the irrational number lies between 2 and 3 is. ($\sqrt{10}$ or $\sqrt{7}$ or 2.5 or $\sqrt{3}$).

B) $\sqrt{10} \simeq$ .... (2.99 or 3.71 or 3 or - 3.2) .

C) The nearest integer to $\sqrt[3]{25}$ is .... (5 or 3 or 2 or 12.5) .

D) The square whose area is 10 cm^2, its side length is .... cm

(5 or -5 or $\sqrt{10}$ or - $\sqrt{10}$).

E) The cube whose volume is 64 cm^3 its side length is ...... cm (8 or 4 or 16 or 64).

6 Draw the number line and label point A which represents $\sqrt{2}$ and Label point B which represents 1 + $\sqrt{2}$, Label point C which represents 1 - $\sqrt{2}$

7 Draw the right triangle ABC at B where AB = 2cm, BC = 3cm, then use the figure to label a point which represents $\sqrt{13}$, and a point which represents -$\sqrt{13}$ on the number line.

The set of the Real numbers R

Think and Discuss

You will learn how

- To define the set of real numbers (R).
- To define the realtion among sets of N, Z, Q, Q', R

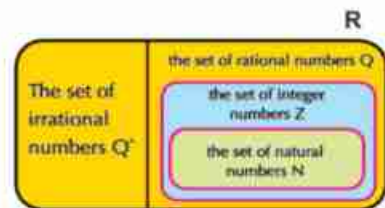
Key terms

- A real number.

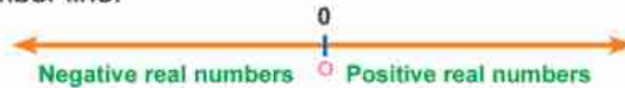
You have learned the set of rational numbers (Q), you have also found that there are other numbers that form the set of irrational number Q' such as $\sqrt{2}$, $\sqrt[3]{2}$, π,... However, the union of these two sets forms a new set called the set of the real numbers, and it is denoted by the symbol R

$$R = Q \cup Q'$$

Look at the opposite Venn diagram, you find that:

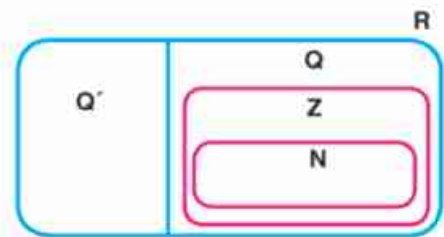
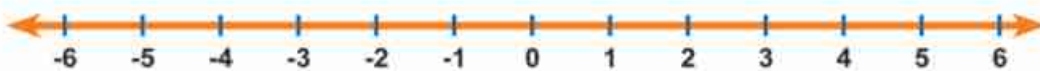
1. $R = Q \cup Q'$
2. Any natural, integer, rational or irrational number is a real number

$N \subset Z \subset Q \subset R$ and so is $Q' \subset R$

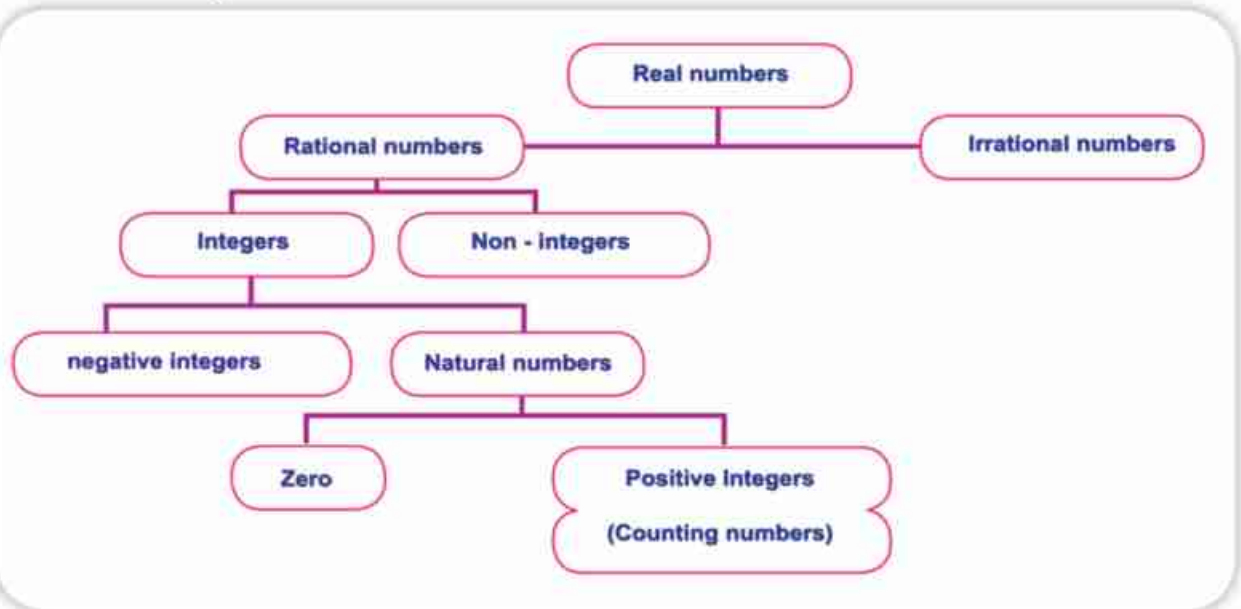
Think Give examples from your own to some real numbers which are rational or irrational numbers.

3. Every real number is represented by one point on the number line.

First: zero is represented by the origin O.

Second: the positive real numbers are represented by all the points On the number line that are located on the right side of O

Third: the negative real numbers are represented by all the points on the number line that are located on the left side of O

1. **Put each of the following numbers in its suitable place on the opposite venn diagram.**
$\frac{1}{2}$, -4 , 9 , $\sqrt{5}$, 0,6 , $\frac{7}{9}$, $\sqrt[3]{-2}$, $\sqrt{16}$, 0 , 5

2. **Label** point A on the number line which represents $\sqrt[3]{-8}$, and point B which represents $\sqrt{9}$, then find the length of $\overline{AB}$.

3. **State if each sentence is true or false:**
   - A. Every natural number is a positive real number.
   - B. Every integer is a real number.

Remark $\sqrt[3]{-1}$ = -1 because -1 × -1 × -1 = -1
While $\sqrt{-1} \notin R$ because there is no real number If multiplyed by it self, the product is -1 .

Discuss with your teacher and classmates: Are there any non- Real number?

Exercises (1 - 4)

1 **Study the previous chart and answer the following by placing (✔) on the true sentence and (✘) on the false sentence:**

- **A** Every natural number is an integer. ( )
- **B** Zero $\in$ The set of rational numbers ( )
- **C** $Z = Z^{+} \cup Z^{-}$ ( )
- **D** Any non-integer number is a rational number ( )

2 **Complete the following table by placing (✔) in the suitable place as shown in the first case:**

Number	natural number	integer number	rational number	irrational number	real number
-5	✗	✓	✓	✗	✓
$\sqrt{2}$					
$1\frac{1}{2}$					
$\sqrt[3]{9}$					
$\|-2\|$					
$-\sqrt{4}$					
$\frac{5}{2}$					
0,3					
$\sqrt{-1}$					

Ordering numbers at R

Think and Discuss

If A, B are two points that belong to the straight line L, and we determined a certain direction as shown by the arrow; then we can say that:

- The point B follows the point A. i.e on its right hand side.
- The point A precedes the point B. i.e on its left hand side.

The same applies for all the points on the straight line.

However, If we know that every point on the straight line represent a real number. We can say that :

the set of real number is an ordered set.:

The properties of order:

1. If x , y are two real numbers represented on the number line by the two points A, B respectively, the ordering relation can be one of the following three cases:

A is congruent to B so x = y	A follows B so $x > y$	A precedes B so $x < y$

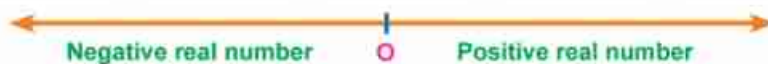
2. If x is a real number represented by the point A on the number line while O is the origin point which represents the zero, then the ordering relation can be one of the following three cases.

A is congruent to O $\therefore x = 0$	A is on the right of O $\therefore x > 0$ then x is a positive real number.	A is on the left of O $\therefore x < 0$ then x is a negative real number.

You will learn how

- To define the ordering relation in R.

Key terms

- ordering relation .
- more than.
- Less than
- Equal to
- Ascending order
- Descending order .

The set of the positive real numbers: $R^+ = \{x : x \in R\ ,\ x > 0\}$

The set of the nagative real numbers: $R^- = \{x : x \in R\ ,\ x < 0\}$

$$R = R^+ \cup \{0\} \cup R^-$$

Remark : The set of non-negative real numbers = $R^+ \cup \{0\} = \{x : x \geqslant 0,\ x \in R\}$

The set of the non - positive real numbers = $R^- \cup \{0\} = \{x : x \leqslant 0,\ x \in R\}$

Example:

Arrange the following numbers ascendingly $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 6, 0, $\sqrt[3]{-1}$

Solution

$6 = \sqrt{36}$, $\sqrt[3]{-1} = -1 = -\sqrt{1}$

The ascending order is from the smallest to the greatest.

$-\sqrt{45}$, $-\sqrt{1}$, 0 , $\sqrt{20}$, $\sqrt{27}$, $\sqrt{36}$

i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0 , $\sqrt{20}$, $\sqrt{27}$, 6.

Exercises (1-5)

1. Arrange the following number descendingly : $\sqrt{62}$, 8 , $-\sqrt{50}$, $\sqrt{70}$

2. If $x \in R$, state whether x is a positive or negative or anything else in each of the following.

   A) $x > 0$ B) $x < 0$ C) $x > |-5|$

3. Prove that $\sqrt{3}$ lies between 1.7, 1.8, then represent $\sqrt{3}$,1.7 and 1.8 on the number line.

4. Find the side length in a square whose area is 5 cm^2, is the side length a rational number?

5. Find the side length in a cube whose volume is 1.728 cm^3, is the side length a rational number?

6. Put the suitable notation(> or < or =)

   A) $\sqrt{5}$ .... 2 B) $\sqrt{7}$ .... 2,6 C) $\sqrt[3]{-24}$ .... -2

   D) $1 + \sqrt{2}$ .... $\sqrt{3}$ E) $\sqrt[3]{8}$ .... $\sqrt{4}$ F) $3 - \sqrt{5}$ .... $\sqrt[3]{-1}$

Think and Discuss

Interval is a subset of the set of real numbers

first: the limited intervals

If $a, b \in R$, $a < b$, then we can define each of:

The closed inteval [a , b]

$[a, b] = \{ x : a \leqslant x \leqslant b,\ x \in R\}$

a b

$[a, b] \subset R$ in which the elements are a, b and all the real numbers between them.

When we draw that interval, we put a shaded circle at each of the two points a and b then, we shade that area between them on the number line.

The open interval]a , b[

$]a, b[= \{ x: a < x < b, x \in R\}$

a b

$]a , b[\subset R$ in which the elements are all the real numbers between the two numbers a, b

When we draw that interval, we put an unshaded circle at each of the two points which represent the two numbers a and b then, we shade that area between them on the number line.

You will learn how

- To define limited intervals.
- To define unlimited intervals.
- To recognize the operations on intervals .

key terms

- Limited interval
- closed interval
- open interval
- half- open interval
- unlimited interval
- union
- intersection
- difference
- complement

Practice

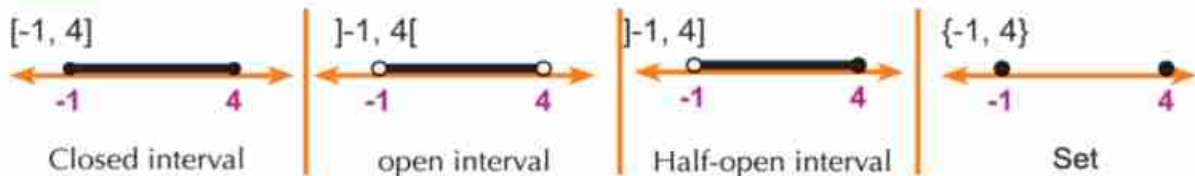
Write down each of [3,5],]3,5[using the description method then represent them on the number line.

Half openor (half closed) intervals

[a , b[	]a , b]
$[a, b[= \{x : a \leqslant x < b , x \in R\}$	$]a , b] = \{ x: a < x \leqslant b , x \in R\}$
$[a, b[\subset R$ where its elements are the number a and all the numbers between a and b.	$]a , b] \subset R$ where its elements are the number b and all the number between a and b .

Write down each of the two intervals: [3 , 5[,]3, 5] using the description method, then represent them on the number line.

Represent each of the following intervals on the number line: [-1, 4] ,]-1, 4[,]-1, 4], {-1, 4}

Solution

Discuss with your teacher and your classmates whether the interval is a finite or an infinite set.

1 Write **down the following sets in the form of intervals, then represent them on the number line:**

A) $X = \{x : 2 < x < 5 , x \in R\}$

B) $X = \{x : -2 \leqslant x < 3 , x \in R\}$

C) $X = \{x : 0 \leqslant x \leqslant 4, x \in R\}$

D) $X = \{x : -3 < x \leqslant -1 , x \in R\}$

2 **Put** The suitable symbol ∈ or ∉ to make each sentence true.

A) 3 .... [-1 , 3[B) -2 ....]-1 , 3[C) $\frac{1}{2}$ ....]0 , 1[

D) $\sqrt{2}$ .... [1 , 2] E) 4 .... [0 , 5[F) $\sqrt[3]{-8}$ .... [-1 , 2]

G) |-5| .... [4 , 6[H) 2.3×10^{-5} ....]0, 1[

3 **Write** down the interval represented by each of the following figures:

A)

C)

B)

D) 0 6

Second: The unlimited intervals

You know that: If the number line of real numbers is expanded on its two direction, we get more positive real numbers at the right direction and more negative real number at the left direction such all those numbers are located on that line.

- The symbol (∞) is read (infinity) and it is more than any imagined real number, ∞ ∉ R
- The symbol (-∞) is read (negative infinity) and it is less then any imagined real number, -∞ ∉ R
- The two symbols ∞ , - ∞ can not be represented by any points on the number line and they are expansions to the number line at its two directions.

If a is a real number, then we can define the following unlimited intervals:

The interval [a , ∞[

[a, ∞[= { x : x ⩾ a , x ∈ R}

That interval represents the number a and all the real numbers which are more than a

The interval]-∞ , a]

]-∞ , a] = { x : x ⩽ a , x ∈ R}

That interval represents the number a and all the real number which are less than a.

Write down each of the following intervals [3, ∞[,]-∞, 3] using the description method, then represent them on the number line.

the interval $]a, \infty[$

$]a,\infty[= \{x : x > a\ ,\ x \in R\}$

That interval represents all the real number which are more them a

the interval $]-\infty\ , a[$

$]-\infty\ ,a[= \{ x : x < a\ ,\ x \in R\}$

that interval represents all the real numbers which are less than a

Write **down the two intervals $]3, \infty[$, $]-\infty, 3[$ using the description method , then represent then on the number line**

Remark : The set of real numbers (R) can be represented in the form of the interval $]-\infty\ , \infty[$

The set of the positive real numbers $R^+ =]\ 0, \infty[$

The set of the negative real numbers $R^- =]-\infty, 0[$

The set of non-negative real numbers = $[0, \infty[$

The set of non-positive real numbers = $]-\infty, 0]$

Practice

1 **Write** **down the following sets in the form of intervals, then represent them on the number line.**

- A $X = \{x : x \geqslant 2\ ,\ x \in R\}$
- B $X = \{x : x < 3\ ,\ x \in R\}$
- C $X = \{x : x > -7\ ,\ x \in R\}$
- D $X = \{x : x \leqslant \sqrt[3]{-8}\ ,\ x \in R\}$
- E the set of all the real numbers more than $|-3|$

2 **Put** **the suitable symbol** $\in$ or $\notin$ or $\subset$ or $\not\subset$ **To make each statement true:**

- A 3 .... $]-\infty, 4[$
- B $[1\ , 2]$ .... $]-1, \infty[$
- C -5 .... $]-\infty, -6[$
- D $]0\ , 2[$ .... $]0, \infty[$
- E 3×10^{10} .... $]3, \infty[$
- G $[-3, 1]$ .... $[\ 2, \infty[$

Operations on intervals

Since all the intervals are subsets of the set of the real number R, The operations of union, intersection, difference and complement can be applied on the intervals. The graphical representation to the intervals on the number line contributes to determine and verify the result of any operation. This can be clarified from the following examples:

Examples

1 **If X = [-2, 3] , Y = [1, 5 [, find the following using the number line:**

A $X \cap Y$ B $X \cup Y$

Solution

A $X \cap Y = [-2, 3] \cap [1, 5[= [1, 3]$

B $X \cup Y = [-2, 3] \cup [1, 5[= [-2, 5[$

2 **If M = [2, ∞[, J =] -2, 3[, find the following using the number line:**

A M - J B $M \cap J$ C $M \cup J$

D $J \cup \{2, 3\}$ E M` F J`

Solution

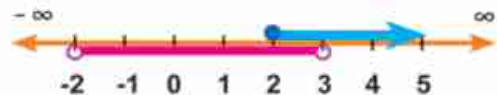
A $M - J = [2, \infty[-]-2, 3[= [3, \infty[$

B $M \cap J = [2, \infty[\cap]-2, 3[= [2, 3[$

C $M \cup J = [2, \infty[\cup]-2, 3[=]-2, \infty[$

D $J \cup \{2, 3\} =]-2, 3[\cup \{2, 3\} =]-2, 3]$

E $M` =]-\infty, 2[$

F $J` =]-\infty, -2] \cup [3, \infty[$

Practice

Put (✓) on the true sentence and (✗) on the false sentence:

A $[-2, 5] - \{2, 5\} =]-2, 5[$

B $]-1, 3] \cup \{-1, 0\} = [-1, 0]$

C $[2, 5] - \{5\} = [2, 5[$

D $[-1, 3] \cap]1, 4[= [1, 3]$

E $[-2, 5[\cup \{1, 5\} = [-2, 5]$

F $[5, \infty[-]-\infty, 5] =]5, \infty[$

Exercises (1-6)

1 **Complete** the following table as shown in the first example:

Interval	Representation by using the descri·tion method	Graphical representation on the number line
[-1 , 2]	$\{x : -1 \leqslant x \leqslant 2, x \in R\}$	-1 0 1 2
[1 , 3[		
]-∞ , 2]		
	$\{x : 0 < x \leqslant 3, x \in R\}$	
	$\{x : x > -1, x \in R\}$	
		-∞ -2 -1 0 1 2 3 ∞
		-∞ 1 ∞
]1, 5[		
	$\{x : x > 0, x \in R\}$	

2 **Complete** using $\in$ or $\notin$:

A. 3 .... [2 , 3]

B. $\sqrt[3]{-1}$ ....]-∞ , 1[

C. 2 .... {1 , 7}

D. $\sqrt{9}$ ....]-3 , ∞[

E. | -2| .... [2 , ∞[

F. 1.3×10^{-5} ⋆ .... R_+

3 **Choose** the correct answer :

A. [2 , 7] - { 2 , 7} = ... ([1 , 6] or ∅ or]2 , 7[or {0})

B. [0 , 5] ∪ [3 , 8[= .... (]3 , 5] or [3 , 5] or [0 , 8] or [0 , 8[)

C. [1 , 5] ∩]-2 , 3] = ... ({1 , 3} or]1 , 3[or [1 , 3] or [1 , 3[)

D.]-1 , 2[- [1 , 4] = ... (]-1 , 1[or {-1 , 1} or]-1 , 1] or [-1 , 1])

4 **If X = [-1 , 4] , Y = [3 , ∞[, Z = {3, 4}, find each of the following using the number line:**

A. X ∪ Y B. X ∩ Y C. X - Y D. X - Z

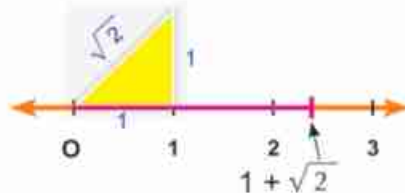
E. Y ∩ Z F. Y - X G. X\`  H. Y\`

Operations on the real numbers

Think and Discuss

First: The properties of adding the real numbers :

You have determined the location of the point X which represents the number $1 + \sqrt{2}$ on the number line. Since it represents the sum of the two real numbers 1 and $\sqrt{2}$ then the sum of every two real numbers is a real number.

i.e, the set of the real numbers R is closed under the operation of addition.

the closure property If $a \in R$, $b \in R$ then $(a + b) \in R$

for example : each of $2 + 3$, $1 + \sqrt{2}$, $-2 + \sqrt{5}$ and $2 + \sqrt[3]{3}$ are real numbers.

The commutative property If $a \in R$, $b \in R$ then $a + b = b + a$

for example : $2 + \sqrt{3} = \sqrt{3} + 2$, $3 - \sqrt{5} = -\sqrt{5} + 3$

The associative property If $a \in R$, $b \in R$, $c \in R$, then $(a + b) + c = a + (b + c) = a + b + c$

for example : $(3 + \sqrt{2}) + 5 = 3 + (\sqrt{2} + 5)$ associative property

$= 3 + (5 + \sqrt{2})$ commutative property

$= 3 + 5 + \sqrt{2}$ associative property

$= 8 + \sqrt{2}$

You will learn how
- To solve operations on the real numbers .
- To define the properties of operations on the real numbers .

key terms
- Closure property.
- Commutative property.
- associative property.
- Additive neutral.
- Additive inverse.
- multiplicative neutral.
- multiplicative inverse.
- distribution of multiplication on addition or subtraction.

Zero is the additive neutral element: If $a \in R$ then $a + 0 = 0 + a = a$

for example : $\sqrt{5} + 0 = 0 + \sqrt{5} = \sqrt{5}$, $-\sqrt[3]{4} + 0 = 0 + (-\sqrt[3]{4}) = -\sqrt[3]{4}$

Each real number has an additive inverse For a number $a \in R$ there is $(-a) \in R$ where $a + (-a) = (-a) + a = \text{zero}$

for example $\sqrt{3} \in R$, has additive inverse $(-\sqrt{3}) \in R$ where
$\sqrt{3} + (-\sqrt{3}) = (-\sqrt{3}) + \sqrt{3} = \text{zero}$.

Practice

1 **Complete the following to have a true sentence:**

A $\sqrt{2} + 5 = 5 +$ ....

B $\sqrt{11} + (-\sqrt{11}) =$ ....

C $7 + \sqrt{3} = 5 +$ (................ + ...)

D the additive inverse for $\sqrt[3]{8}$ is ....

E the additive inverse for $(1 - \sqrt{2})$ is ....

F $\sqrt{3} + (-\sqrt{3}) =$ ....

G $7 + \sqrt{5} - 3 =$ ....

H $(4 + \sqrt{7}) + (3 - \sqrt{7}) =$ ....

I If $a \in R$, $b \in R$, then $a - b$ means the sum of the number a and .... of the number b.

J If $a \in N$, $b \in Q$, $c \in R$, then $(a + b + c) \in$ ....

2 **Discuss the following with your teacher and classmates, then give examples:**

A Is subtraction a commutative operation in R?

B Is subtraction an associative operation in R?

Second: The properties of multiplying the real numbers

The closure property If $a \in R$, $b \in R$ then $a \times b \in R$

the set of real number is closed under the operation of multiplication.

i.e the product of multiplying every two real number is a real number.

for example : $5 \times \sqrt{2} = 5\sqrt{2} \in R$, $\sqrt{3} \times \sqrt{3} = 3 \in R$

$-2 \times \sqrt[3]{5} = -2\sqrt[3]{5} \in R$, $\frac{2}{3} \times \pi = \frac{2}{3}\pi \in R$

$2\sqrt{3} \times \sqrt{3} = 6 \in R$, $2\sqrt{3} \times 5 = 10\sqrt{3} \in R$

Commutative property If $a \in R$ and $b \in R$, then $a . b = b . a$

for example : $\sqrt{2} \times 3 = 3 \times \sqrt{2} = 3\sqrt{2}$

The associative property If $a \in R$, $b \in R$, $c \in R$, then :

$(a . b) . c = a . (b . c) = a . b . c$

for example : $\sqrt{2} \times (5 \times \sqrt{2}) = (\sqrt{2} \times 5) \times \sqrt{2} = (5 \times \sqrt{2}) \times \sqrt{2}$

$= 5 \times \sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$

One is the multiplicative neutral If $a \in R$, then $a . 1 = 1 . a = a$

for example : $2\sqrt{5} \times 1 = 1 \times 2\sqrt{5} = 2\sqrt{5}$

Every real number ≠ 0 has a multiplicative inverse If $a \neq 0$

It exist an real number $\frac{1}{a}$ such that

$a . \frac{1}{a} = \frac{1}{a} . a = 1$ (1 is the neutral element of multiplication)

for example: the multiplicative inverse for $\frac{\sqrt{3}}{2}$ is $\frac{2}{\sqrt{3}}$

where $\frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$

Remark : $\frac{a}{b} = a \times \frac{1}{b}$, $b \neq 0$

i.e. $\frac{a}{b} = a \times$ the multiplicative inverse of b.

Discuss with your teacher: is the division operation commutative in R? Is the division operation associative in R?

Examples

Write down each of the following numbers $\frac{6}{\sqrt{2}}$, $-\frac{5}{\sqrt{3}}$, $\frac{15}{2\sqrt{5}}$ where the denominator is an integer .

Solution

Note that the multiplicative neutral is 1 and It can be written in the form $\frac{\sqrt{2}}{\sqrt{2}}$ or $\frac{\sqrt{3}}{\sqrt{3}}$ or $\frac{\sqrt{5}}{\sqrt{5}}$ or ...

$$\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = \frac{3\sqrt{2}}{1} = 3\sqrt{2}$$

$$-\frac{5}{\sqrt{3}} = -\frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{5\sqrt{3}}{3}$$

$$\frac{15}{2\sqrt{5}} = \frac{15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{2 \times 5} = \frac{3\sqrt{5}}{2}$$

Practice

1 Complete the following to have a true sentence:

A. $\sqrt{2} + \sqrt{2} + \sqrt{2} = \quad\quad \times \sqrt{2} =$

B. $3 \times \sqrt{5} = \sqrt{5} \times$

C. $\sqrt{7} \times \sqrt{7} =$

D. $2\sqrt{5} \times 3\sqrt{5} =$

E. The multiplicative neutral in R is the number

F. The multiplicative inverse for $\frac{3}{\sqrt{2}}$ is

2 Write each of the following numbers such that the denominator is an integer:

A. $\frac{15}{\sqrt{6}}$

B. $\frac{8}{3\sqrt{2}}$

C. $-\frac{6}{\sqrt{3}}$

D. $\frac{25}{2\sqrt{10}}$

Distribution of multiplication on addition

For any three real numbers a, b, c.

$a \times (b + c) = (a \times b) + (a \times c) = a\,b + a\,c$

$(a + b) \times c = (a \times c) + (b \times c) = a\,c + b\,c$

Examples

1 **Simplify the following to the simplest form .**

A $2\sqrt{5}\left(3+\sqrt{5}\right)$ B $\left(\sqrt{2}+5\right)\left(3+\sqrt{2}\right)$

C $\left(2-3\sqrt{5}\right)^2$

Solution

A $2\sqrt{5}\left(3+\sqrt{5}\right) = 2\sqrt{5} \times 3 + 2\sqrt{5} \times \sqrt{5}$

$= 2 \times 3 \times \sqrt{5} + 2 \times 5 = 6\sqrt{5} + 10$

B $\left(\sqrt{2}+5\right)\left(3+\sqrt{2}\right) = \sqrt{2}\left(3+\sqrt{2}\right) + 5\left(3+\sqrt{2}\right)$

$= \sqrt{2} \times 3 + \sqrt{2} \times \sqrt{2} + 5 \times 3 + 5 \times \sqrt{2}$

$= 3\sqrt{2} + 2 + 15 + 5\sqrt{2}$

$= 3\sqrt{2} + 17 + 5\sqrt{2} = 8\sqrt{2} + 17$

C $\left(2-3\sqrt{5}\right)^2 = (2)^2 + 2 \times 2 \times -3\sqrt{5} + \left(-3\sqrt{5}\right)^2$

$= 4 - 12\sqrt{5} + 9 \times 5$

$= 49 - 12\sqrt{5}$

2 **Give an estimation to the result of $(3+\sqrt{5}) \times (1+\sqrt{8})$, then check your answer using the calculator.**

Solution

First: The estimate of $\sqrt{5}$ is 2 $\therefore (3+\sqrt{5})$ the estimate of $3 + 2 = 5$

the estimate of $\sqrt{8}$ is 3 $\therefore (1+\sqrt{8})$ the estimate of $1 + 3 = 4$

$\therefore (3+\sqrt{5})(1+\sqrt{8})$ the estimate of $5 \times 4 = 20$

Second: when we use the calculator to find $(3+\sqrt{5}) \times (1+\sqrt{8})$

We find that the result is 20.0459

Therefore, the estimate is reasonable.

Exercises (1-7)

1 **Choose the correct sentence:**

A) $2\sqrt{3} + 3\sqrt{3} =$ .... $(5\sqrt{6}$ or $5\sqrt{3}$ or $6\sqrt{3}$ or $5\sqrt[3]{3})$

B) $\sqrt[3]{5} + \sqrt[3]{5} =$ .... $(\sqrt[3]{10}$ or 5 or $2\sqrt[3]{5}$ or $5\sqrt[3]{5})$

C) $5 + 7\sqrt{2} - 4 + \sqrt{2} =$ .... $(15$ or $1 + 7\sqrt{2}$ or $1 + 8\sqrt{2}$ or $1 + 6\sqrt{2})$

D) $-2\sqrt{3} \times \sqrt{3} =$ .... $(-6$ or $-2\sqrt{3}$ or $2\sqrt{3}$ or $6)$

E) $\frac{6}{\sqrt{3}} =$ .... $(\sqrt{2}$ or 2 or $2\sqrt{3}$ or $6\sqrt{3})$

F) $(2\sqrt[3]{5})^3 =$ .... $(10$ or 20 or $4\sqrt[3]{5}$ or $40)$

2 **Simplify to the simplest form:**

A) $\sqrt{2}(5 + \sqrt{2})$

B) $\sqrt{7}(\sqrt{7} + 2)$

C) $-\sqrt{3}(-5 - \sqrt{3})$

E) $(\sqrt{2} + 1)(\sqrt{2} - 1)$

3 **Write each of the following numbers where the denomirator is an integer:**

A) $\frac{10}{\sqrt{5}}$

B) $\frac{8}{\sqrt{6}}$

C) $\frac{6}{2\sqrt{3}}$

D) $\frac{\sqrt{2} + 3}{\sqrt{2}}$

4 **Simplify to the simplest form:**

A) $2\sqrt{3} + 5 + \sqrt{3} - 6$

B) $2\sqrt{7} - 3\sqrt{2} + \sqrt{7} + 5\sqrt{7}$

C) $(\sqrt{3} + 2)(\sqrt{3} - 1)$

D) $\sqrt{5}(3 - \sqrt{5}) - 2(1 + \sqrt{5})$

5 IF $a = \sqrt{3} + 2$, $b = \sqrt{3} - 2$ **Find the value of each of the following :**

A) $a + b$

B) $a - b$

C) $a\,b$

6 **If** $X = \sqrt{15} + 2$, $Y = 4 - \sqrt[3]{25}$ **estimate the value :**

A) X , Y

B) $X \times Y$

C) $X + Y$

Check the reasonablility of each value using your calculator.

Operations on the square roots

Think and Discuss

If a , b are two non-negative real numbers, then

First: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

For example : $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

$\sqrt{2} \times \sqrt{10} = \sqrt{2 \times 10} = \sqrt{20}$

$\sqrt{15} \times \sqrt{5} = \sqrt{15 \times 5} = \sqrt{75}$

$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

For example : $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

Second: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where $b \neq 0$

For example : $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{1}{3}\sqrt{5}$

$\sqrt{\frac{16}{3}} = \frac{\sqrt{16}}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

Third: $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ $b \neq 0$

For example : $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

$\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

You will learn how

- To conduct operations on the square roots.
- To multiply two conjugates.

key terms

- Square root.
- Two conjugates numbers .

Examples

1 Simplify to the simplest form $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

Solution

$$\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}} = \sqrt{16 \times 2} - \sqrt{36 \times 2} + 6 \times \frac{\sqrt{1}}{\sqrt{2}}$$

$$= \sqrt{16} \times \sqrt{2} - \sqrt{36} \times \sqrt{2} + 6 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = \sqrt{2}$$

2 If $x = 2\sqrt{5} - 1$, $Y = 2 + \sqrt{5}$ find the value of $x^2 + y^2$

Solution

$$x^2 = (2\sqrt{5} - 1)^2 = (2\sqrt{5})^2 - 4\sqrt{5} + 1$$

$$= 4 \times 5 - 4\sqrt{5} + 1 = 21 - 4\sqrt{5}$$

$$y^2 = \left(2 + \sqrt{5}\right)^2 = 4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$$

$$x^2 + y^2 = 21 - 4\sqrt{5} + 9 + 4\sqrt{5} = 30$$

Practice

1 put each of the following in the form of a $\sqrt{b}$ where a and b are integers, b is the least possible value:

A $\sqrt{28}$ B $\sqrt{75}$ C $\sqrt{54}$

D $\sqrt{1000}$ E $2\sqrt{72}$ F $\frac{1}{3}\sqrt{162}$

2 Simplify to the simplest form:

A $2\sqrt{18} \times 3\sqrt{2}$ B $\sqrt{5} \times 2\sqrt{10}$ C $3\sqrt{7} \times 2\sqrt{28}$

D $\sqrt{50} + \sqrt{8}$ E $\sqrt{20} - \sqrt{45}$ F $\sqrt{27} + 5\sqrt{18} - \sqrt{300}$

3 **Find** the value of $X + Y$, $X \times Y$ in each of the following cases:

A $x = 3 + \sqrt{5}$, $y = 1 - \sqrt{5}$ B $x = \sqrt{3} - \sqrt{2}$, $y = \sqrt{3} + \sqrt{2}$

C $x = 5 - 3\sqrt{2}$, $y = 5 - 3\sqrt{2}$

The two conjugate numbers

If a and b are two positive rational numbers.

Then each of the two number $(\sqrt{a} + \sqrt{b})$, $(\sqrt{a} - \sqrt{b})$ is a conjugate to the other one.

then, their sum is = $2\sqrt{a}$ twice the first term

and their product is = $(\sqrt{a} + \sqrt{b})$, $(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

= The square of the first term - The square of the second term

The product of two conjugates is always a rational number

If we have a real number whose denominator is written in the form $(\sqrt{a} \pm \sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

Practice

Complete

A $\sqrt{5} + \sqrt{2}$ their conjugate (...............) and their product is (...............)

B $5 - \sqrt{3}$ their conjugate (...............) and their product is (...............)

C $2\sqrt{3} + \sqrt{2}$ their conjugate (...............) and their product is (...............)

Examples

1 **Given** $x = \frac{8}{\sqrt{5} - \sqrt{3}}$, $y = \frac{2 - \sqrt{3}}{2 + \sqrt{3}}$

Write both of X and Y where the denominator is a rational number, then find X + Y

Solution

$$x = \frac{8}{\sqrt{5} - \sqrt{3}} = \frac{8}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

$$= \frac{8(\sqrt{5} + \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{8(\sqrt{5} + \sqrt{3})}{5 - 3} = 4(\sqrt{5} + \sqrt{3})$$

$$y = \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{(2-\sqrt{3})^2}{4-3} = \frac{4-4\sqrt{3}+3}{1} = 7-4\sqrt{3}$$

$$x + y = 4\sqrt{5} + 4\sqrt{3} + 7 - 4\sqrt{3} = 4\sqrt{5} + 7$$

2 Given $x = \frac{4}{\sqrt{7}-\sqrt{3}}$, $y = \sqrt{7} - \sqrt{3}$,

prove that **x and y are conjugates, then find the values of:**

$x^2 - 2xy + y^2$, $(x - y)^2$. What do you observe?

Solution

$$x = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3} = \sqrt{7}+\sqrt{3}$$

$y = \sqrt{7} - \sqrt{3}$ $\therefore$ x , y (two conjugate numbers)

$$x^2 - 2xy + y^2 = (\sqrt{7}+\sqrt{3})^2 - 2(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) + (\sqrt{7}-\sqrt{3})^2$$

$$= (7 + 2\sqrt{21} + 3) - 2(7 - 3) + (7 - 2\sqrt{21} + 3)$$

$$= 10 + 2\sqrt{21} - 8 + 10 - 2\sqrt{21}$$

$$= 12$$

$$(x - y)^2 = [(\sqrt{7}+\sqrt{3}) - (\sqrt{7}-\sqrt{3})]^2$$

$$= [\sqrt{7} + \sqrt{3} - \sqrt{7} + \sqrt{3}]^2 = (2\sqrt{3})^2$$

$$= 4 \times 3 = 12$$

Remark : $x^2 - 2xy + y^2 = (x - y)^2$

Practice

In the previous example, find the value of each of the following:

- **A** $(x + y)$
- **B** $(x - y)$
- **C** $(x + y)(x - y)$
- **D** $x^2 - y^2$

What do you observe?

Exercises (1-8)

1 **Choose the correct answer :**

A) $\sqrt{50} - \sqrt{18} - \sqrt{2} =$ .... ($\sqrt{30}$ or $\sqrt{2}$ or 2 or $2\sqrt{2}$)

B) $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) =$ .... (2 or 12 or $2\sqrt{7}$ or $-2\sqrt{5}$)

C) $(\sqrt{8} + \sqrt{2})^2 =$ .... ($\sqrt{10}$ or 10 or 18 or $\sqrt{18}$)

D) The multiplicative inverse of $\frac{\sqrt{3}}{6}$ is .... ($-\frac{\sqrt{3}}{6}$ or $6\sqrt{3}$ or $2\sqrt{3}$ or $-2\sqrt{3}$)

E) The next number in the pattern: $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$ is .... ($\sqrt{50}$ or $\sqrt{75}$ or $\sqrt{60}$ or $\sqrt{90}$)

2 **Complete the following to get a correct sentence:**

A) If $X = 3 + \sqrt{2}$ then the conjugate is .... and the product of the number by its conjugate is ....

B) The multiplicative inverse for $(\sqrt{3} + \sqrt{2})$ in its simplest form is ....

C) **Think** If $x^2 = 5$, then $(x + \sqrt{5})^2 =$ .... or ....

D) **Think** If $\frac{1}{X} = \sqrt{5} - 2$, then the value of X in its simplest form is ....

E) $3\sqrt{2} + \sqrt{8} - \sqrt{18} =$ ....

3 Simplify to the simplest form $2\sqrt{5} + 6\sqrt{\frac{1}{3}} - \sqrt{12} - 5\sqrt{\frac{1}{5}}$

4 If $x = \frac{4}{\sqrt{7} - \sqrt{3}}$, $y = \frac{4}{\sqrt{7} - \sqrt{3}}$. Find the value of $x^2 y^2$

5 If $A = \sqrt{3} + \sqrt{2}$, $B = \frac{1}{\sqrt{3} + \sqrt{2}}$ .Find the value of $A^2 - B^2$ in its simplest form.

6 If $x = \sqrt{5} + \sqrt{2}$, $y = \sqrt{5} - \sqrt{2}$.

Find the value of $\frac{x + y}{x y - 1}$ in its simplest form

7 If $x = \sqrt{7} + \sqrt{5}$, $y = \frac{2}{X}$

find the value of $\frac{x + y}{x y}$ in its simplest form

Operations on the cube roots

Think and Discuss

You will learn how

- To carry operations on the cube roots.

key terms

- Cube root.

For any two real numbers a, b :

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a \times b}$

For example : $\sqrt[3]{5} \times \sqrt[3]{2} = \sqrt[3]{5 \times 2} = \sqrt[3]{10}$

$\sqrt[3]{3} \times \sqrt[3]{-4} = \sqrt[3]{3 \times -4} = \sqrt[3]{-12}$

2 $\sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$

For example : $\sqrt[3]{40} = \sqrt[3]{8 \times 5} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$

$\sqrt[3]{-128} = \sqrt[3]{-64 \times 2} = \sqrt[3]{-64} \times \sqrt[3]{2} = -4\sqrt[3]{2}$

3 $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$ where $b \neq o$, $a, b \in R$

For example : $\frac{\sqrt[3]{12}}{\sqrt[3]{3}} = \sqrt[3]{\frac{12}{3}} = \sqrt[3]{4}$

4 $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$ where $b \neq o$, $a, b \in R$

For example : $\sqrt[3]{\frac{3}{2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2}}$

 Think : If we multiply both the numerator and the denominator by $\sqrt[3]{4}$, then find the product in its simplest form

Examples :

Simplify to the simplest form:

A $\sqrt[3]{54} + 8\sqrt[3]{\frac{-1}{4}} + 5\sqrt[3]{16}$

B $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}}$

Solution

A $\sqrt[3]{54} + 8\sqrt[3]{\frac{-1}{4}} + 5\sqrt[3]{16} = \sqrt[3]{27 \times 2} + 8\sqrt[3]{\frac{-1}{4} \times \frac{2}{2}} + 5\sqrt[3]{8 \times 2}$

$= \sqrt[3]{27} \times \sqrt[3]{2} + 8\frac{\sqrt[3]{-2}}{\sqrt[3]{8}} + 5 \times \sqrt[3]{8} \times \sqrt[3]{2}$

$= 3\sqrt[3]{2} + \frac{8 \times (-\sqrt[3]{2})}{2} + 5 \times 2 \times \sqrt[3]{2}$

$= 3\sqrt[3]{2} - 4\sqrt[3]{2} + 10\sqrt[3]{2} = 9\sqrt[3]{2}$

B $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}} = \sqrt[3]{24} - 6\sqrt[3]{\frac{125}{9}} = \sqrt[3]{8 \times 3} - 6 \times \sqrt[3]{\frac{125}{9} \times \frac{3}{3}}$

$= \sqrt[3]{8} \times \sqrt[3]{3} - 6 \times \frac{5\sqrt[3]{3}}{3} = 2\sqrt[3]{3} - 2\sqrt[3]{10} = -8\sqrt[3]{3}$

Exercises (1-9)

1 Put each of the following in the form a $\sqrt[3]{b}$ where a and b are two integers, b is the least positive possible value:

A $\sqrt[3]{54}$ B $\sqrt[3]{-1000}$ C $\sqrt[3]{128}$

D $\sqrt[3]{-2160}$ E $\sqrt[3]{1715}$ F $\sqrt[3]{686}$

2 **Find** the result for each of the following in the simplest form:

A $\sqrt[3]{125} - \sqrt[3]{24}$ B $\sqrt[3]{250} - \sqrt[3]{128}$ C $\sqrt[3]{\frac{2}{5}} \times \sqrt[3]{\frac{4}{25}}$

D $\sqrt[3]{\frac{3}{4}} \div \sqrt[3]{\frac{2}{9}}$ E $\frac{1}{2}\sqrt[3]{56} - \sqrt[3]{\frac{7}{27}}$ F $\frac{1}{2}\sqrt[3]{10} \times 6\sqrt[3]{100}$

3 If $a = \sqrt[3]{5} + 1$, $b = \sqrt[3]{5} - 1$ find the value of each of the following

A $(a - b)^5$ B $(a + b)^3$

4 **Prove that:**

A $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = \text{zero}$

B $\sqrt[3]{54} \times \sqrt[3]{16} \div (\sqrt[3]{4} \times 6) = 1$

Applications on the real numbers

Think and Discuss

You will learn how

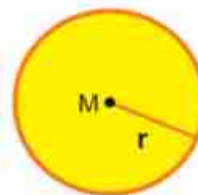
- To solve applications on square and cube roots.

key terms

- Circle
- Cuboid
- Cube
- Right circular cylinder
- Sphere

The circle:

Circumference of a circle = 2 π r length unit

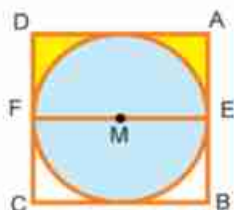
where r is the length of the radius in a circle,
π is the (approximate ratio).

Examples

1 **Find** the circumference of a circle whose area is 38.5 cm^2 $(\pi = \frac{22}{7})$

Solution

The area of the circle = π r^2

$38.5 = \frac{22}{7} r^2 \qquad \therefore r^2 = \frac{38.5 \times 7}{22} = \frac{49}{4}$

$\therefore r = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5$ cm

2 In the opposite figure, the circle M is inside the square ABCD. If the area of the yellow sector is $10\frac{5}{7}$ cm^2, find the perimeter of the sector$(\pi = \frac{22}{7})$

Solution

We suppose that the length of the raidus in a Circle = r

$\therefore$ The side length of the square = 2r

The area of the yellow color = the area of the rectangle AEFD - the area of semi circle

$$10\frac{5}{7} = r \times 2r - \frac{1}{2} \times \frac{22}{7} r^2$$

$$\frac{75}{7} = 2r^2 - \frac{11}{7} r^2 = \frac{3}{7} r^2$$

$\therefore r^2 = 25 \quad \therefore r = 5$ cm

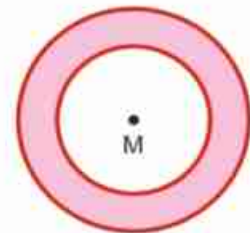
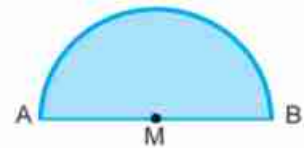
The perimeter of the yellow sectors = (AE + AD + DF) + $\frac{1}{2}$ the circumference of the circle

$$= (5 + 10 + 5) + \frac{1}{2} \times 2 \times \frac{22}{7} \times 5 = 35\frac{5}{7} \text{ cm}$$

Practice

1. A circle whose area is 64π cm^2. Find the length of its radius, then find its circumference approximating it to the nearest integer ($\pi = 3.14$).
2. In the figure opposite: $\overline{AB}$ is the diameter of a semi circle. If the area of that region is 12.32cm^2. Find the circumference of that figure.

3. In the opposite figure: there are two circles have the same center "concentric" of center M. If the lengths of their radii are 3cm and 5cm. Find the area and the circumference of the colored region in the terms of π.

M

The cuboid

It is a body whose six faces are of a rectangular shape such that every two opposite faces are congruent.:

If the lengths of its edges were x, y, z, then:

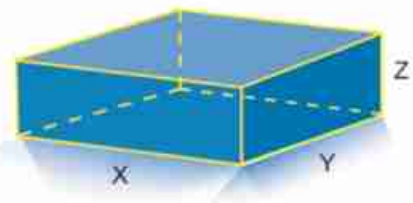
The lateral area = the perimeter of the base × the height

The lateral area = 2 (x + y) × z square unit

The lateral area = the lateral area + 2 × the area of the base

The total area = 2 (xy + yz + Xz) square unit

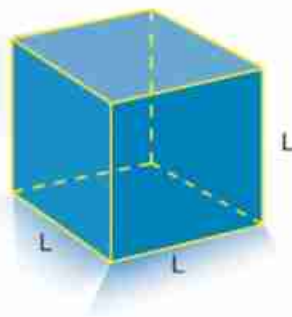
The volume of the cuboid = the area of the base × the height

The volume of the cuboid = x × y × z cubic unit

A special case : the cube

It is a cuboid whose edges are equal in length. If the length of one edge = L length unit, then:

The area of each face = L^2 square unit

The lateral area of each face = $4L^2$ square unit

The total area = $6L^2$ square unit, the volume of the cube = L^3 cubic unit

Examples

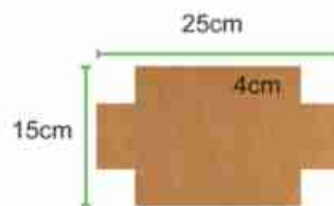
Find the total area of a cube whose volume is 125 cm^3

Solution

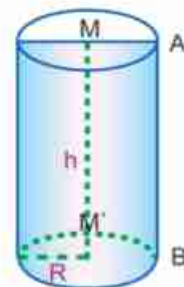
The volume of the cube = L^3 $\therefore 125 = L^3$ $\therefore L = \sqrt[3]{125} = 5cm$

The total area = $6L^2 = 6 \times (5)^2 = 150\ cm^2$

Practice

1. Find the total area of a cuboid whose volume is 720cm^3 and height 5cm with a squared shape base.
2. Which is more in volume: A cube of 294 cm^2 area or a cuboid with the following dimensions: $7\sqrt{2}$, $5\sqrt{2}$, 5 cm.
3. A rectangular hard piece of paper has a length of 25 cm and a width of 15 cm. A square whose side = 4 cm was cut from each of its four corners. Then, the projected parts were folded to form a shape of a cuboid. Find the volume and the total area of that cuboid.

The right circular cylinder :

It is a body that has two parallel congruent bases each is a circular shaped surface, while its lateral surface is a curved surface called cylindrical surface.

- If M, M` are the bases of the cylinder, then M M` is the height of cylinder.

Let's think If A $\in$ the circle M, B $\in$ the circle M`,

$\overline{AB} \parallel \overline{MM`}$

- Then, if we cut the lateral cylindrical surface at AB and we stretch That surface, we get the surface of the rectangle A B B` A`

Then, AB = height of cylinder, A A` = the perimeter of the base of the cylinder.

The area of the rectangle A B B` A` = the lateral area of the cylinder

The lateral area of the cylinder = the perimeter of the base × height = $2\pi r h$ (square unit)

the total area of the cylinder = area of lateral surface + sum of the areas of the two bases

$= 2\pi r h + 2\pi r^2$ (square unit)

the volume of the cylinder = base area × height = $\pi r^2 h$ (cubic unit)

Example

A piece of paper has shape of a rectangle ABCD in which AB = 10cm, BC = 44cm. It was folded to form a right circular cylinder such that $\overline{AB}$ is congruent to $\overline{DC}$. Find the volume of the resulted cylinder. ($\pi = \frac{22}{7}$).

Solution

The perimeter of the cylinder base = 44 cm.

$2\pi r = 44$

$2 \times \frac{22}{7} r = 44$

$\therefore r = 7$cm

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10$

$= 1540 \text{ cm}^3$

Practice

1. Find the volume and the total area of a right circular cylinder in which the length of base radius = 14 cm and the height is 20 cm .

2. Find the total area of a right circular cylinder of volume 7536 cm^3 and height 24 cm ($\pi = 3.14$)
3. Which is more in volume: a right circular cylinder of radius 7cm and height 10 cm or a cube whose edge length is equal to 11cm

The sphere:

It is a body of curved surface in which the points have the same distance (r) from a constant point inside it (the center of the sphere)..

If the sphere is cut by a plane passing by its center, then the resulted section is a circle whose center is the center of a sphere where its radius is the radius of a sphere (r).

Volume of the sphere = $\frac{4}{3}\pi r^3$ cubic units.

area of the sphere = $4\pi r^2$ square units.

Examples

The volume of the sphere is 562.5π cm^3 . Find its surface area.

Solution

the volume of sphere = $\frac{4}{3}\pi r^3$

$562.5\pi = \frac{4}{3} \times \pi r^3$

$\therefore r^3 = 562.5 \times \frac{3}{4} = 421.875$

$r = \sqrt[3]{421.875} = 7.5$cm

the surface area of sphere = $4\pi r^2 = 4 \times \pi (7.5)^2 = 225\pi$ cm^2

Practice

Find the volume and the surface area of a sphere whose diameter is 4.2cm ($\pi = \frac{22}{7}$)

Exercises (1-10)

1 **Choose the correct answer**

A. The lateral area of a right circular cylinder whose base diameter length b and height h is .... (πb^2h, πbh, $2\pi bh$, πbh^2).

B. The volume of a sphere whose diameter length is 6cm = ... cm^3 (288 , 12π , 36π , 288π)

C. The edge length of a cube whose volume is $2\sqrt{2}$ cm^3 = ... cm. ($\sqrt{2}$, 2, 8, 1.5)

D. The radius length of a right circular cylinder whose volume is 40π cm^3 and height 10cm = ... cm (5 , 3 , 2 , 1).

E. The volume of a cuboid whose dimensions are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$ cm is .... (6 , 36 , $6\sqrt{6}$, $18\sqrt{2}$).

2 **Complete**

A. The radius length of a sphere whose volume is $\frac{9}{2}\pi$ cm^3 is .... cm.

B. A right circular cylinder whose base radius length r and height h, then its lateral area is ..... and its volume is .....

C. The total area of a cube of edge length = 4 cm is ..... cm^2.

D. The lateral area of the cuboid = ...

3 A sphere with volume 36π cm^3 is placed inside a cube, if the sphere touches the cube's six faces. Find

A. the radius of the sphere.

B. the volume of the cube.

4 A metal sphere with diameter 6 cm has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height.

5 Find the height of a right circular cylinder whose height is equal to its base radius and its volume is 72π cm^3.

6 A hollow metal sphere with internal radius 2.1 cm and external radius 3.5 cm. Find its mass to the nearest gram given that the mass of a cubic centimeter of such a metal is 20 gm (π $\frac{22}{7}$)

Solving Equations and Inequalities of first degree in one variable in R

Think and Discuss

You will learn how

- To solve equation of first degree in one variable in R.
- To solve inequalities of first degree in one variable

key terms

- -equation
- degree of an equation.
- Inequality
- degree of an inequality
- Solution of an equation
- Solution of an inequality

First:Solving Equations of first degree in one variable in R

We know that: The equation $3X - 2 = 4$ is called an equation of first degree where the exponent of the (unknown) variable X is 1. To solve that equation in R

$3x - 2 = 4$ **By adding 2 to the sides of the equation**

$3x = 6$ **(we can multiply by the multiplicative inverse of the coefficient of X)**

$\frac{1}{3} \times 3x = \frac{1}{3} \times 6$

$\therefore x = 2$

i.e the solution set { 2 }

This solution can be graphed on the number line as shown in the figure opposite .

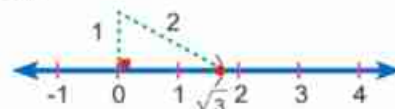
Examples

1 **Find the solution set of the equation $\sqrt{3}\,x - 1 = 2$, in R, then graph the solution on the number line.**

Solution

$\sqrt{3}\,x - 1 = 2 \qquad \therefore \sqrt{3}\,x = 3$

$\therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \qquad \therefore x = \sqrt{3} \in \mathbb{R}$

The solution set is $\{\sqrt{3}\}$

This solution can be graphed on the number line as shown in the figure opposite .

Find The solution set for the equation $x + \sqrt{2} = 1$, in R, then graph the solution on the number line.

Solution

$x + \sqrt{2} = 1 \qquad \therefore x = 1 - \sqrt{2} \in R$

This can be graphed on the number as shown in the figure opposite .

Practice

1 **Find the solution set for the following equations in R, then graph the solution on the line number.**

A $5x + 6 = 1$	B $2x + 4 = 3$	C $2x - 3 = 4$
D $x + 5 = 0$	E $\sqrt{2}\, x - 1 = 1$	F $x - 1 = \sqrt{5}$

Second : Solving inequalities of the first degree in one variable in R, graphing the solution on the number line.

The following properties are used to solve the inequality in R. The solution set is written in the form of an interval

If A , B، C were real number where A< B, then:

1	$A + C < B + C$.	addition property.
2	If $C > 0$ then $A \times C < B \times C$.	property of multiplication by a positive real number
3	If $C < 0$ then $A \times C > B \times C$.	property of multiplication by negative real number.

Examples

  Find the solution set for the inequality $2x - 1 \geq 5$ in R and represent the solution set graphically.

Solution

By adding 1 to the sides of the inequality it becomes $2x \geq 6$

by multiplying the side of the in equality by $(\frac{1}{2} > 0)$ $\quad x \geq 3$

$\therefore$ The solution set in R is $[3 , \infty[$

and it is graphed by green color ray on the number line.

Find the solution set for the inequality $5 - 3x > 11$, in R, then represent the solution graphically.

Solution

By adding (-5) to the sides of the inequality then $-3x > 6$

by multiplying the sides of the inequality by $(-\frac{1}{3} < 0)$ we get :

$\therefore x < -2$

i.e., the solution set in R is est $]-\infty, -2[$

and it is represented by the green color ray on the number line.

  Find the solution set for the in equality $-3 \leqslant 2x - 1 < 5$ in R and represent the solution graphically.

Solution

by adding (1) to the sides of the inequality $-3 + 1 \leqslant 2x - 1 + 1 < 5 + 1$

Namely, $-2 \leqslant 2x < 6$, and by multiplying the sides of the inequality by $(\frac{1}{2} > 0)$ $-1 \leqslant x < 3$

$\therefore$ the solution set in R is $[-1, 3[$ and it is graphed on the number line by the green color.

in example What is the solution set for the inequality in N?

What is the solution set for the inequality in Z?

Exercises (1-11)

  Complete the following to have a true sentence where $X \in R$

A. If $5x < 15$, then x ....

B. If $x - 3 \geqslant 4$, then x ....

C. If $-2x \leqslant 3$, then x ....

D. If $1 - x > 4$, then x ....

E. If $\sqrt{2}\, x \leqslant 4$, then x ....

2 **Find** The solution set for each of the following inequalities in R in the form of intervals, then graph the solution on the number line.:

A $3x - 1 < 5$ B $2x + 5 \geqslant 3$

C $2x + 3 \leqslant 1$ D $5 - x > 3$

E $1 - 5x < 6$ F $\frac{1}{2}x + 1 \leqslant 2$

3 **Find** the solution set for each of the following inequalities in R in the form of an interval, then represent the solution on the number line.

A $-1 \leqslant 2x + 1 < 5$ B $-5 \leqslant 2x - 3 \leqslant 1$

C $-3 \leqslant 4x - 7 \leqslant 5$ D $4 < 3x + 4 < 7$

E $1 < 5 - x \leqslant 3$ F $1 \leqslant 3 - 2x < 5$

4 **Find** the solution set for each of the following inequalities in R, write it in the form of an interval, then represent the solution on the number line.:

A $-3 \leqslant -x < 3$ B $|-3| < 2x - 1 < 5$

C $\sqrt[3]{-8} \leqslant x + 1 \leqslant \sqrt{9}$ D $5 < 3 - x \leqslant 3^2$

General Exercises

1 **Complete** **the following to have a true sentence**:

A $\sqrt{9} + \sqrt[3]{-8} =$ ....

B A vessel that has the shape of a cub whose capacity is 8 liters, the length of the edge of its interior face is .... cm.

C The solution set for the equation $x^2 + 9 = 0$ in R is ....

D $(\sqrt{3} + \sqrt{2})^2 + (\sqrt{3} - \sqrt{2})^2 =$ ....

E A rectangle of dimensions are $(\sqrt{5} + 1)$, $(\sqrt{5} - 1)$ cm has an area of ....cm².

F $\sqrt[3]{54} - \sqrt[3]{-16} = \sqrt[3]{\dots}$

G $[-1, 5] -]-1, 5[=$ ....

H The solution set for the equation $\sqrt{2}x - 1 = 3$ .in R is.........

I A sphere whose diameter is equal to 6 L (length unit) has a volume of .... (cubic unit).

J $|\sqrt[3]{-125}| = \sqrt{\dots}$

2 **Find** **the solution set for each of the following inequalities in R, in the form of an interval, then represent the solution on the number line.:**

A $5x - 3 < 2x + 9$

B $3 - 4x \leqslant x - 2$

C $x \leqslant 2x - 1 \leqslant x + 3$

D $x - 1 < 3x - 1 \leqslant x + 1$

E $4x \leqslant 5x + 2 < 4x + 3$

F $5x + 7 > 6x > 5x$

3 If $x = \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - \sqrt{5}}$ prove that $x + \frac{1}{x} = 22$

4 **Find** in the simplest form: $\sqrt[3]{54} + 4\sqrt[3]{\frac{1}{4}} - \sqrt[3]{-2}$

5 **Find** the total area of a right circular cylinder of volume 72π cm^3, and height 8 cm.

6 **Find** the following using the number line $[3, 6[\cap [4, 7[$

7 If $x = \frac{5\sqrt{2} + 3\sqrt{5}}{\sqrt{5}}$, $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$ Find the value

A $x^2 + y^2$

B $x\,y$

and prove that $x^2 + y^2 = 38\,x\,y$

8 If $X = \sqrt[3]{5} + 2$, $y = \sqrt[3]{5} - 2$ Find the value of $(x + y)^3 + (x - y)^3$.

9 If $x = \sqrt{5} - \sqrt{3}$, $y = \frac{2}{\sqrt{5} - \sqrt{3}}$, Find the value of $(X^2 + 2xy + y^2)$

10 If $A = \sqrt{3} + \sqrt{2}$, $B = \sqrt{3} - \sqrt{2}$,

find the value of $(A^2 - AB + B^2)$

11 If $x = \frac{3\sqrt{5} + 5\sqrt{2}}{\sqrt{5}}$, $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$

Prove that $\frac{x^2 + y^2}{x\,y} = 38$

Technology

Find: $\sqrt[3]{27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0.125}$

Open microsoft office Excel and record the shown numbers in cells A1, D1 and B1.

To Find the cube root of cell A1, write the formula A1^(1/3) in the cell F1, then Enter... the result is 3.

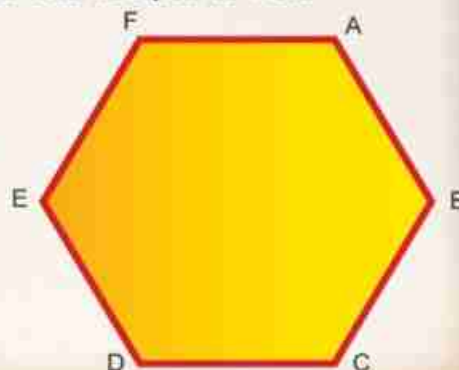
To find the square root of cell B1, write the formula B1^(1/2) in the cell H2, then Enter... the result is 3.5.

To find the cube root of cell J1, write the formula D1^(1/3) in the cell J2, then Enter... the result is 0.5.

Write down the sum of F2+H2+J2 in cell L2, after you do a click on = then the sum is 1.

Activity

Activity **Draw a regular hexagon ABCDEF whose side is equal to 4 cm.**

1. **Find the measure of its interior angle.**
2. Draw its diagonals $\overline{AD}$, $\overline{BE}$, $\overline{CF}$ then deduce the length of each diagonal without measuring them by the ruler.
3. draw a circle that passes by its vertices.
4. Find its area.

Unit test

1 **Complete** **to get a true sentence :**

A. $]-3, 2] \cap R^- =$ ....

B. The multiplicative inverse for $-\frac{\sqrt{2}}{6}$ is ....

C. $\sqrt{5}, \sqrt{20}, \sqrt{45}, \sqrt{80},$ .... Complete in the same pattern.

D. If $x = \sqrt[3]{3} + 7$, $y = \sqrt[3]{3} - 7$ then $(x + y)^3 =$ ....

E. The circle of 20X cm circumference has an area of .... π cm^2

2 **Choose** **the correct sentence from Parentheses:**

A. a cube of 64 cm^3 has a lateral area of =...cm^2 (4 or 8 or 64 or 96)

B. $\sqrt{12} - \sqrt{3} =$ (3 or $\sqrt{3}$ or $2\sqrt{3}$ or $3\sqrt{3}$)

C. The multiplicative inverse for $\frac{-\sqrt{6}}{12}$ is ($\frac{12}{\sqrt{6}}$ or $\frac{\sqrt{6}}{12}$ or $-2\sqrt{6}$ or $2\sqrt{6}$)

D. $\sqrt[3]{54} + \sqrt[3]{-2} =$ ($\sqrt[3]{52}$ or $\sqrt[3]{2}$ or $2\sqrt[3]{2}$ or $4\sqrt[3]{2}$)

E. $[-3, 4] - \{-3, 5\} =$ ($]-3, 4[$ or $]-3, 4]$ or $]-3, 5[$ or $[-3, 4[$)

3 Reduce to the simplest from $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$

4 a lead cuboid in which the lengths of its faces are 77cm, 24cm and 21cm. It was melted to form a sphere. Find the radius of that sphere. ($\pi = \frac{22}{7}$)

5 If $A = \frac{4}{\sqrt{7} - \sqrt{3}}$, $B = \frac{4}{\sqrt{7} + \sqrt{3}}$ find the value of $\frac{A - B}{AB}$

6 Using the number line find $]-1, 3] \cup [0, 5[$ in the form of an interval

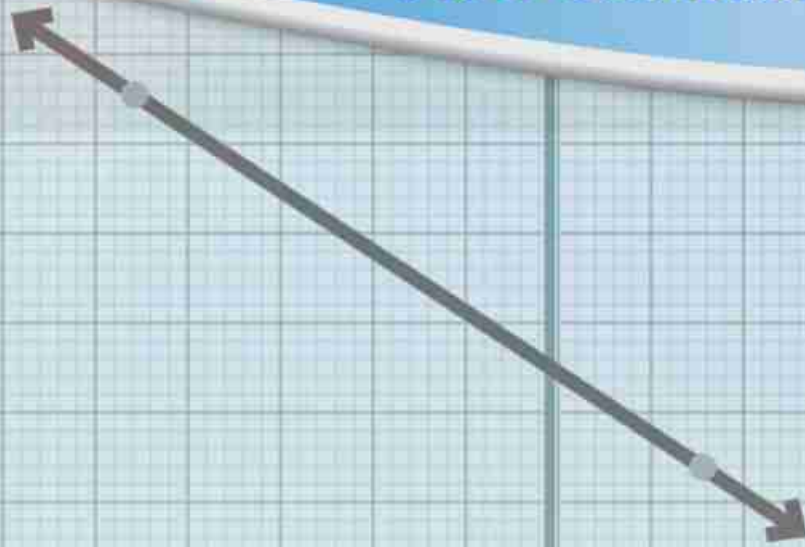
7 Find the lateral area for a right circular cylinder of volume 924cm^3 and height 6cm ($\pi = \frac{22}{7}$).

8 If $x = \sqrt{10} + 2$, $y = \sqrt[3]{26} - 1$. Give an estimate for the product of $X \times Y$ and use the calculator to find the difference between your estimate and the exact answer.

9 Find the Solution set in R and graph the solution on the number line.

A. $1 < 2x + 3 \leq 9$

B. $x + 2\sqrt{3} = 3$

UNIT TWO
2
Relation Between
Two Variables

Linear Relation of two variables

Think and Discuss

You will learn how

- The linear Relations of two variables
- To graph the linear relations of two variables

key terms

- Variable
- Relation
- Linear equation

A person has some bills of LE 50 and LE 20. He bought an electrical apparatus for LE 390.

Think: **How many bills of each type does he give to the seller?**

Suppose : x represents the number of fifties bills, then the value of what he has of these bills is L.E 50x, y represents the number of Twenties bills, then the value of what he has of these bills is L.E 20y.

Required is to know: x and y that verify the equation:

$$50x + 20y = 390$$

This relation represents a linear equation in two variables. Dividing both sides over 10 produces the following equivalent equation:

$$5x + 2y = 39$$

$$\therefore y = \frac{39 - 5x}{2}$$

Note that : x and y are natural numbers. Therefore, x should be an odd number.

The following table can be created to know the different possibilities of giving bills to the seller: a bill of L.E50 and 17 bills of L.E 20, or 3 bills of L.E 50 and 12 bills of L.E 20, or 5 bills of L.E 50 and 7 bills of L.E 20, or 7 bills of 50 and 2 bills of L.E 20.

x	y	(x , y)
1	17	(1 , 17)
3	12	(3 , 12)
5	7	(5 , 7)
7	2	(7 , 2)
9	negative	refused

1. A person has some bills of L.E 5 and some of L.E20. He bought some goods from a shopping center for L.E75. What are the different possibilities of paying this amount in the two types of bills which he has?
2. The perimeter of an isosceles triangle is 19cm. What are the different possible lengths of its sides? Side length $\in Z_+$

Remember : The sum of the lengths of any two sides of a triangle is greater than the length of the third side .

The Relation of two variables

$a x + by = c$ where $a \neq 0 , b \neq 0$ is called a linear relation of two variable **x** and **y** and can be described by a set of ordered pairs (**x**, **y**) verifying this relation.

Example:

Refer to the relation $2x - y = 1$

If x = 1, ∴ y = 1 ∴ (1 , 1) satisfies the relation

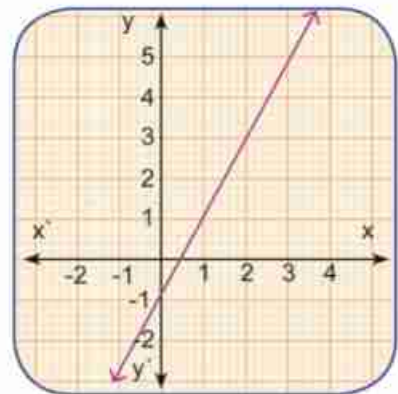
If x = 0, ∴ y = -1 ∴ (0 , -1) satisfies the relation

If x = 3, ∴ y = 5 ∴ (3 , 5) satisfies the relation

If x = -1, ∴ y = -3 ∴ (-1 , -3) satisfies the relation

Thus, there are an infinite number of ordered pairs satisfying the relation.

Note that:

- The linear relation $2x - y = 1$, can be represented graphically by using any of the ordered pairs obtained before.
- Each point ∈ the straight line (in red) is represented by an ordered pair whose elements satisfy the linear relation $2x - y = 1$.

Practice

1. **Find four ordered pairs satisfy each linear relation and represent it graphically:**

   A. $x + y = 3$ B. $x - 2y = 5$ C. $y = 2$ D. $x = 1$

2. Find the value of b, where (-3, 2) satisfies the relation $3x + b\,y = 1$.

3. Find the value of k, where (k, 2k) satisfies the relation $x + y = 15$.

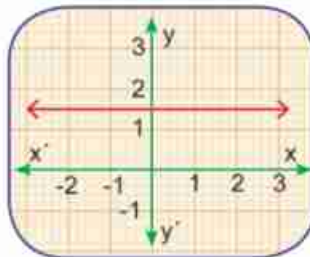
Graphing the Relation of two Variables

The relation **$ax + by = c$** where a and b or both are not equal zero.
is called a linear relation of two variables x and y and can be represented graphically by a straight line.

for **a = 0**

The relation is represented by a straight line parallel to x-axis.

Example : $2y = 3$

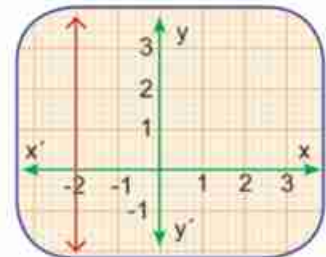
i.e. : $y = \frac{3}{2}$
is represented by the red line which passes through the point $(0.\ \frac{3}{2})$ and is parallel to x-axis.

Special case:

the relation y = 0 represents the x-axis

for **b = 0**

The relation is represented by a straight line parallel to y-axis.

Example : $x = -2$

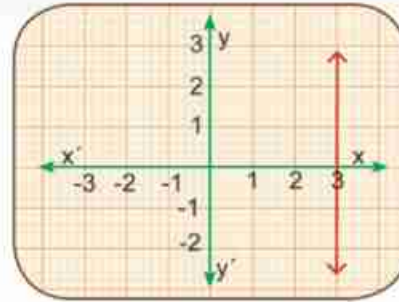
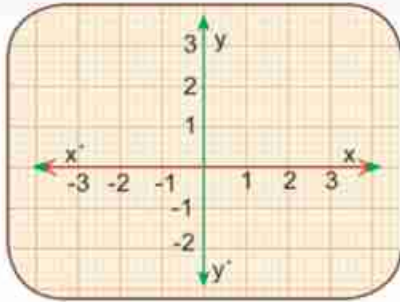
is represented by the red line which passes through the point (-2, 0) and is parallel to y-axis.

Special case:

the relation x = 0 represents the y-axis.

Practice

1. **Graph each relation of the following:**

   A. $2x = 5$ B. $y + 1 = 0$

2 **Find the relation that is represented by the red line in each figure below:**

Example :

Graph the relation: $x + 2y = 3$

Solution

Choose some ordered pairs that satisfy the relation:

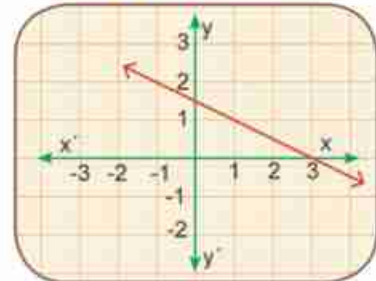
Example : For $y = 2$ $\therefore x = -1$ $(-1, 2)$ **satisfies the relation**

$y = 0$ $\therefore x = 3$ $(3, 0)$ **satisfies the relation**

$y = -1$ $\therefore x = 5$ $(5, -1)$ **satisfies the relation**

and so on ....

The following table lists these data:

x	-1	3	5	0
y	2	0	-1	$\frac{3}{2}$

The red line represents this relation.

Discuss with your teacher:

1. What happens to the value of y when increasing the value of x?
2. When does the line representing the relation $ax + by = c$ pass through the origin 0?

Exercises (2 – 1)

1 **Graph each of the following:**

A. $x + y = 2$ B. $2x - y = 3$

2 Graph the relation $2x + 3y = 6$. If the straight line representing this relation intersects the x-axis in point A and the y-axis in point B. Find the area of the triangle OAB, where 0 is the origin.

The Slope of a line and real-life Applications

Think and Discuss

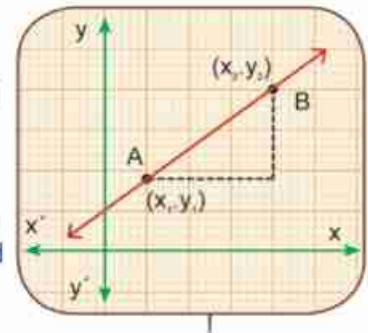
You will learn how

- The slope of a line .
- Real-life applications on the slope of a line.

key terms

- Slope.
- Positive slope.
- Negative slope.
- Zero-slope.
- Undefined slope.

When observing the motion of a point on a straight line from the location A (x_1, y_1) to the location B (x_2, y_2), where $x_2 > x_1$ and A, B $\in$ line, then:

1. the change in x-coordinate $= x_2 - x_1$, and is called the horizontal change.
2. the change in y-coordinate $= y_2 - y_1$ is called the vertical change and may be positive, negative or zero.

The slope of a line = $\dfrac{\text{change in y-coordinate}}{\text{change in x-coordinate}} = \dfrac{\text{vertical change}}{\text{horizontal change}}$

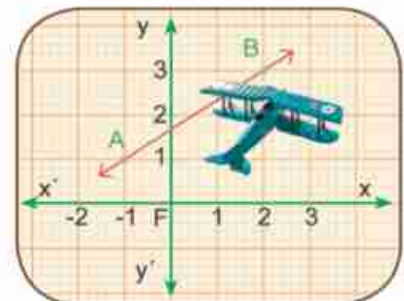
$$S = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_2 \neq x_1$$

In the following examples you will learn different cases of the vertical change $(y_2 - y_1)$:

Example (1)

If: A (-1, 1) and B (2, 3),

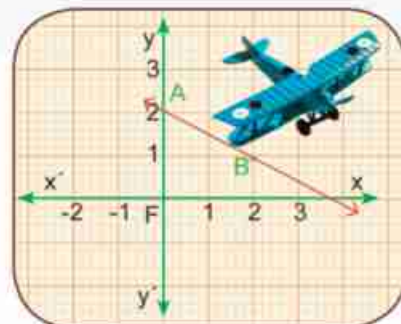
then: the slope of $\overleftrightarrow{AB}$

$= \dfrac{3 - 1}{2 - (-1)} = \dfrac{2}{3}$

Note that :

1. The point A moves on the line upwards to the point B.
2. $y_2 > y_1$
3. The slope of the line is positive.

Example (2) :

If: A (0, 2), B (2, 1);

then: the slope of $\overleftrightarrow{AB} = \frac{1-2}{2-0} = -\frac{1}{2}$

Not that :

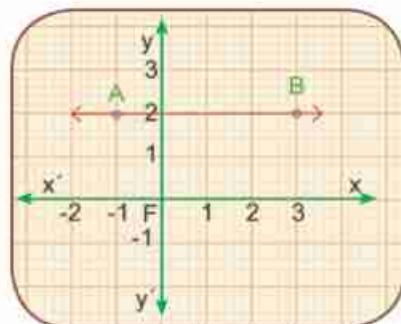
The point A moves on the line downwards to the point B

2. $y_2 < y_1$
3. The slope of the line is negative.

Example (3) :

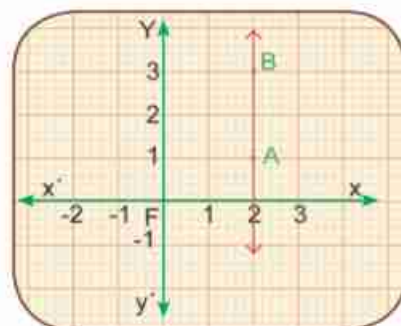
If: A (-1, 2) and B (3, 2),

then: the slope of the line

$\overleftrightarrow{AB} = \frac{2-2}{3-(-1)} = \frac{0}{4} = 0$

Not that :

1. The point A moves horizontally to point B.
2. $y_2 = y_1$
3. The slope of the line = zero

Example (4) :

If: A = (2, 1) and B(2, 3) then: we can not calculate the slope. Because the definition of the slope is conditioned to have a change in the x-coordinate i.e. $x_2 - x_1 \neq 0$

Not that :

1. The point A moves vertically to point B.
2. $x_2 = x_1$
3. The slope of the line is an underfined number.

Practice :

1. Find the slope of the straight line $\overleftrightarrow{AB}$ in each of the following cases:
   - A) A (1, 2), B (5, 0).
   - B) A (2, -1), B (4, -1).
   - C) A (-1, 3), B (2, 1).
   - D) A (3, -1), B (3, 2).

2. Find the slope of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$, where A (2, -1), B (3, 2), and C (4, 5) and represent each line graphically. What do you observe?

3. **Choose the true answer:**

First: The following table shows the relation between x and y as follows:
(y = x + 4 or y = x + 1 or y = 2x -1 or y = 3x -2)

x	1	2	3	4	5
y	1	3	5	7	9

Second: If (2, -5) satisfies the relation 3x –y + c = 0, then c = (1, -1, 11, -11)

Third: (3, 2) does not satisfy the relation (y +x = 5, 3y –x=3, y + x = 7, y – x = 1)

Fourth: An irrigation machine consumes 2.47Litres of diesels to work for 3 hours. If the machine works for 10 hours, it consumes.... litres. (7.2, 8, 8.4, 9.6)

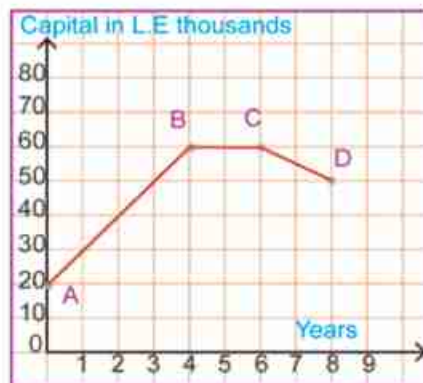
4. **Find the slope of the line $\overleftrightarrow{AB}$, where A(-1, 3) and B (2, 5).**
   Is the point c (8, 1) ∈ $\overleftrightarrow{AB}$?

Real-life Applications on the slope of a line.

Application (1) :

The opposite figure shows capital change of a company during 8 years.

A) Find the slope of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$
What is the meaning of each?

B) Find the starting capital of the company.

Solution

A (0 , 20), B (4 , 60), C (6 , 60) , D (8 , 50)

First: The slope of = $\overleftrightarrow{AB} = \frac{60 - 20}{4 - 0} = 10$, shows the increasing of the capital during the first four years with a rate of 10 thousand pound.

The slope of $\overleftrightarrow{BC} = \frac{60 - 60}{6 - 4} = 0$, means that the capital was constant during the fifth and sixth years.

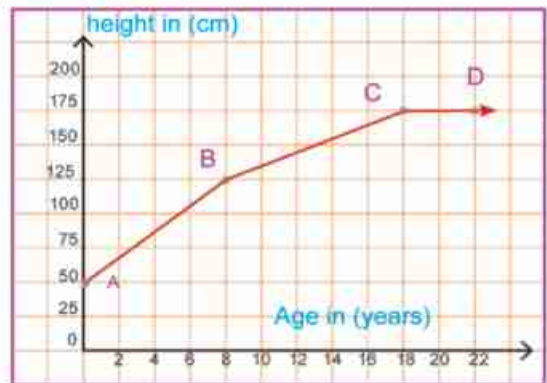
The slope of $\overleftrightarrow{CD} = \frac{50 - 60}{8 - 6} = -5$ shows the decreasing of thecapital during the last two years with a rate of 5 Thousand pound.

Second: Starting Capital = the y-coordinate of the point A = LE 20,000

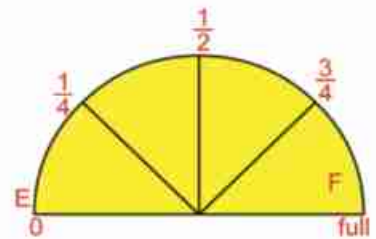
Practice:

The opposite figure shows the relation between the height of a person (in cm) and his age (in years).

First: Find the slope of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

What is the meaning of each?

Second: Calculate the difference between the height of this person as he was 8 years old and his height as he was 30 years old.

Application (2) :

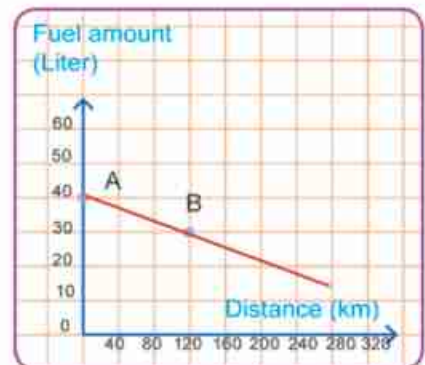
Hazem filled up the 40 Litres tank of his car. As covering a distance of 120 km, the fuel gage shows the rest of fuel is $\frac{3}{4}$ of the tank. Draw a diagram to show the relation between the amount of fuel in the tank and coverd distance (This relation is linear).Calculate the coverd distance as the tank is totally getting empty.

Solution

On the starting point: A (0, 40)

traveled distance — the amount of used fuel

After covering 120 km B = (120, 30)

The slope of $\overleftrightarrow{AB} = \frac{30 - 40}{120 - 0} = \frac{-1}{12}$

This slope means the fuel amount decreases with a rate of 1L per 12 km ,which means 1L is enough to cover a distance of 12 km.

The coverd distance that make the tank empty = $\frac{\text{Fuel Amount}}{\text{Decreasing Rate}} = \frac{40}{\frac{1}{12}}$

$= 40 \times \frac{12}{1} = 480\text{km}.$

Note that : $\overrightarrow{AB}$ intersects the distance-axis in the point (480, 0) **which gives the required distance.**

Exercises (2 – 2)

1 **Complete to make a true statement:**

A Let A (1, 3) and B (2, 1). Then the slope of $\overleftrightarrow{AB}$ = ...

B If (-1, 5) satisties the relation. $3x + ky = 7$, then k = ...

C The slope of any line parallel to x-axis = ...

D The slope of any line parallel to y - axis = ...

E If A, B and C are collinear points. Then the slope of $\overleftrightarrow{AB}$ = the slope of ....

2 Essam has 10 bills of LE 5 and other bills of LE 20 . He bought some goods from a shopping center for LE 65. Determine the different possibilities to pay this amount of money. Find the relation and graph it.

3 The selling price of a computer table is LE 100 and its chair is LE 50. If the store sells in one week with LE 500. What are the represented expectations to the number of computer tables and chairs?

Represent the relation graphically.

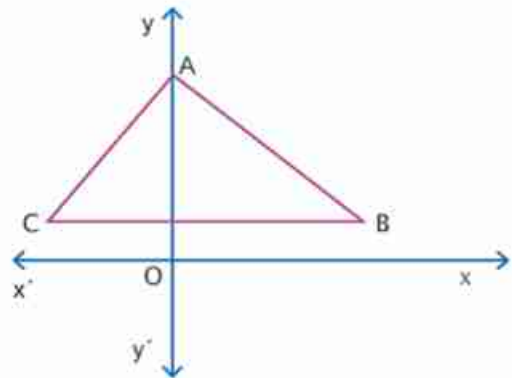
4 **In the opposite figure ABC is a triangle. Complete by using one of the following words:**

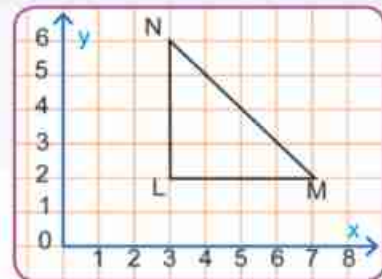
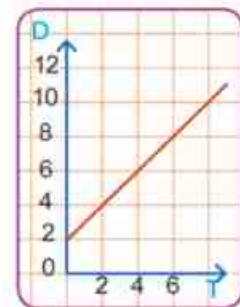
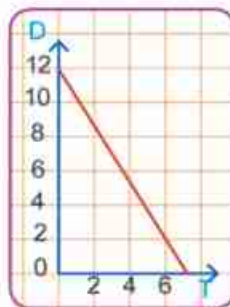
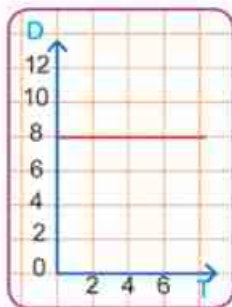
(**positive, negative, zero, undefined**)

A The slope of $\overleftrightarrow{AB}$ ....

B The slope of $\overleftrightarrow{BC}$ ....

C The slope of $\overleftrightarrow{AO}$ ....

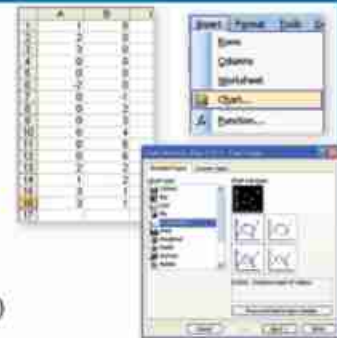
D slope of $\overleftrightarrow{AC}$ ....

5. **In the opposite figure:**

LMN is a right angled triangle at L, where m ($\angle$ M) = 45° , given that L (3, 2) and M (7, 2). Find the coordinates of N and calculate the slope of $\overleftrightarrow{MN}$.

6. **The following diagrams shows the relation between the coverd distance (in m) and the elapsed time (in sec) of an object. Determine the position of the object at the starting motion and its position after 6 seconds when t = 6 sec. Find the slope of the line in each case, and state what it represents.**

Technology:

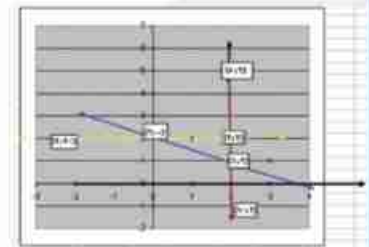
1. open micro soft office Excel to draw the two axes x-y then write the shown number as in fig(1) in the first column A and column B.

2. Do a mouse click to shade the two columns then from the menue INSERT choose CHART as in fig (2) then XY SCATTER as in fig (3) then press NEXT and FINISH, the x-y axes appear.

3. Do a mouse click on from the menue of drawing downward of the page EXCEL and determine the values of dots as shown in fig (4)

4. Do a mouse click on

A. Draw a straight line passes through (2, 1) and (0, 2) then the slope is equal to (2 - 1) \ (0 - 2) which is equal to $\frac{-1}{2}$ of the blue line.

B. Draw a straight line passes through (2, 2) and (-2, 2), then the slope is equal to (2 - 2)\(-2 - 2) which is equal to zero. i.e. the line is parallel to the x axis- the yellow line.

C. Draw a straight line passes through (2, -1) and (2, 5), then the slope is equal to (5-(-1)) \ (2-2), then the slope is undefined, i.e. the line is parallel to the y-axis - the red line.

Activity

The opposite diagram shows the relation between the coverd distance (in km) and elapsed time (in h) of two trains A and B over the distance between two train stations. Use the diagram to find:

- **A** The distance between the two train stations.
- **B** The elapsed time of each train.
- **C** The average speed of each train.
- **D** The meaning of the horizontal segment in the diagram of train A.

- The averege speed = $\frac{\text{The coverd distance}}{\text{Total elapsed time needed to coverd this distance}}$

Unit Test

1 Choose the true answer:

A Which of the following ordered pairs satisfies the relation $2x + y = 5$

((-1, 3), (1, 3), (3, 1), (2, 2))

B Which of the following relations is the relation illustrated in the table.

X	3	4	5
Y	10	13	16

($y = x + 7$, $y = x - 7$, $y = 3x + 1$, $y = x + 1$)

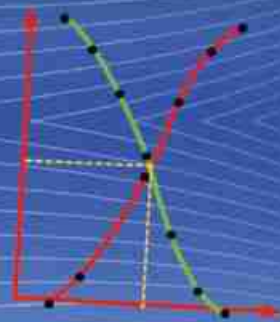
C Let A (3, 5)and B (5, -1), then the slope of $\overleftrightarrow{AB}$ = ($-\frac{1}{3}$, -3, 3, $\frac{1}{3}$)

D The line represented by the relation $3x + 8y = 24$, intersects the y-axis at the point ((0, 8), (8, 0), (0, 3), (3, 0))

2 Let A (2, -1), B (10, 3) and C (2, 3). Find the slope of $\overleftrightarrow{AB}$. $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$

Draw the triangle ABC on a square grid, then tell the type of the trinagle according to its angles.

3 Atef filled up the 50 L-tank of his car. As coverd a distance of 100km, the fuel gage shows the rest of fuel is $\frac{4}{5}$ of the tank. Draw a diagram to show the relation between the amount of fuel remaining in the tank and the distance coverd. What is the coverd distance as the tank is getting totally empty?

UNIT THREE
3
Statistics

Collecting and Organizing data

Think and Discuss

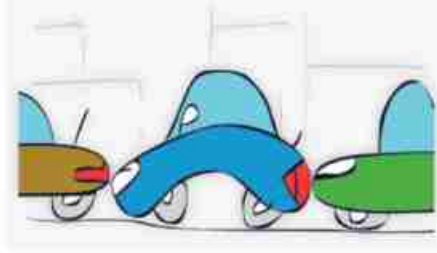
You will learn how

- To collect and organize data
- Using frequency tables with sets

key terms

- Collecting data
- Organizing data.
- Frequency table with sets

If you study the traffic jam problem and its possible solutions:

- What are the sources of your data?
- How can you collect data about such a problem?
- What are the statistical methods you will use to analyze the data?
- Can you explain the results you collected?
- What do you suggest to solve that problem and improve traffic fluidity?

Collecting data

Let's work together Cooperate with your classmates on collecting data from their sources through distribution of roles:

A **Group 1:** Collects primary data about the problem under discussion through a survey that asks about (the means of transportation - Roads conditions - time of traffic jam - Existence of traffic signs - existence of security).

B **Group 2:** Collects secondary data about the problem under discussion from the traffic reports - the internet - the mass media).

C **Group 3:** Observes the crowdest roads, the drivers` behavior and their obedience to traffic rules the pedestrians` commitment to the virtues to the road as well as crossing the roads at safe places.

Organizing and Analyzing data

Cooperate with your classmates on making afrequancy table that represents the means of transportation used by your classmates..

Means of transportation	Subway	bus	Private Car	Taxi	bicycle	on foot	total
Frequency	....	....	....	....	....	....	....

Determine the most used means of transportation (The mode)

1. Is that means suitable? does it help solving the traffic jam problem? why?

2. What do you suggest to solve this problem according to the results you have collected?

Organizing data and representing them in frequency tables

Example

Below are the scores of 30 students in an examination

7	10	7	4	5	8	6	7	13	12
2	9	11	12	11	9	15	12	13	9
5	14	19	3	9	14	3	13	8	17

Required: forming a frequency table with sets that represents that data .

Solution

To form a frequency table with sets, follow the following steps:

First: find the highest and the lowest values of the collected data?

let the previous collected data be X

then: $X = \{x : 2 \leq x \leq 19\}$

i.e: X values begins with 2 and ends in 19

i.e: the range = the highest value - the lowest value = 19 – 2 = 17

Second: divide set X into a number of separate subsets each of them is equal in range.

let them be 6 sets. ∴ The range of the set = $\frac{17}{6}$ i.e approximated to 3

Third: the subsets are as follow.

The first set	2 –
The second set	5 –

the third set	8 –
The Fourth set	11 –

and so on

Remark : 2- means the set of data greater than or equal to 2 and less than 5 and so on.

Fourth: Record the data in the following table:

Set	tally	frequency
2 –	////	4
5 –	~~////~~ /	6
8 –	~~////~~ //	7
11 –	~~////~~ ///	8
14 –	///	3
17 –	//	2
Total		30

Fifth: Delete the tally column from the table to get the frequency table with sets. It can be written either verticaly or horizontaly. The following is the horizontal form of the table:

Sets	2 –	5 –	8 –	11 –	14 –	17 –	total
Frequency	4	6	7	8	3	2	30

Exercises (3 – 1)

1 **Below are the weekly wages of 40 workers in a factory in L.E**

47	71	36	94	54	64	87	89	62	57
51	61	44	52	70	66	56	32	69	36
79	48	77	90	65	99	96	67	60	55
95	75	81	84	78	38	49	94	48	59

Required : Form a frequency table with sets (use the subsets: 30-, 40-, 50-, ...90-). What is the set with the highest frequency? what is the set with lowest frequency?

2 **The following are the scores of 30 students in a monthly math exam.**

25	35	40	20	30	37	40	33	22	38
35	36	28	37	39	28	32	26	29	37
23	34	35	36	29	38	40	35	37	31

Required:

- **A** Form a frequency table with sets for these scores.
- **B** Find the total number of excellent students. The excellence rate is 36 marks or more.

3 **The following table shows the days-off which 40 workers got during a year.**

15	30	26	14	28	13	25	14	27	11
24	16	21	16	15	22	21	17	21	29
26	21	15	20	30	24	20	20	15	26
29	30	20	27	22	26	22	28	30	15

Required:

- **A** Form a frequency table with sets to represent the data above.
- **B** Find the number of workers who got more than 20 days-off a year long.

The Ascending and Descending Cumulative Frequency Table and Their Graphical Representation

Think and Discuss

You will learn how

- To Form both ascending and descending cumulative frequency tables.
- To represent both ascending and descending cumulative frequency tables graphically.

key terms

- The frequency distribution.
- The frequency table.
- The ascending cumulative frequency table.
- The descending cumulative frequency table.
- The ascending cumulative frequency curve.
- The descending cumulative frequency curve.

First: Ascending cumulative frequency table and its graphical representation.

Examples

The following table shows the frequency distribution for the heights of 100 students in a school in centimeters.

Tall (sets) in c.m	115–	120–	125–	130–	135–	140–	145–	Total
Number of students (frequency)	8	12	19	23	18	13	7	100

1. How many students are with height less than 115cm?
2. How many students are with height less than 135cm?
3. How many students are with height less than 145cm?

Form the ascending cumulative frequency table for these data and represent them graphically.

Solution

- Are there students with height less than 115c.m? **No**
- Are there students with height less than 135c.m? How many? **yes, 62 student.**
- How can you calculate the number of students with height less than 145 cm? **Add the number of students in the sets of height less than the set 145.**

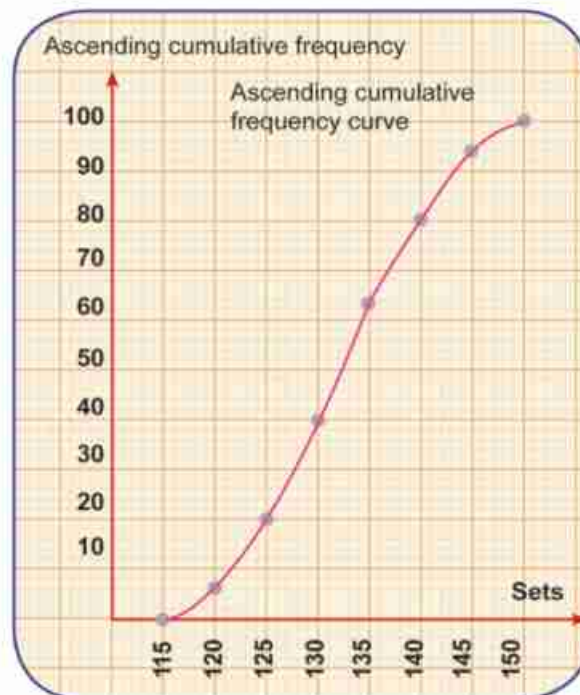
Now, to answer the previous questions in an easier way, form an ascending cumulative frequency table as follows:

Upper boundaries of sets	Ascending cumulative frequency
Less than 115	0
Less than 120	0 + 8 = 8
Less than 125	8 + 12 = 20
Less than 130	20 + 19 = 39
Less than 135	39 + 23 = 62
Less than 140	62 + 18 = 80
Less than 145	80 + 13 = 93
Less than 150	93 + 7 = 100

i.e.

ascending cumulative frequency table	
Upper boundaries of sets	Ascending cumulative frequency
Less than 115	zero
Less than 120	8
Less than 125	20
Less than 130	39
Less than 135	62
Less than 140	80
Less than 145	93
Less than 150	100

To represent the ascending cumulative frequency table graphically:

1. Specify the horizontal axis to the sets and the vertical axis to the ascending cumulative frequency
2. Choose a drawing scale to draw the vertical axis such that the ascending cumulative frequency axis can hold the number of elements in a set
3. Represent the ascending cumulative frequency for each set and draw its line graph successively.

Second: The descending cumulative frequency table and its graphical representation. :

Of the previous frequency distribution which shows the heights of 100 students in a school in centimeters.

Find: The number of students with heights of 150cm and more..

The number of students with heights of 140cm and more..

The number of students with heights of 125cm and more..

Form the descending cumulative frequency table and represent it graphically..

Solution

There are no students with heights of 150cm and more .

The number of students with heights of 140cm and more is 7 + 13 = 20 students.

The number of students with heights of 125cm and more is

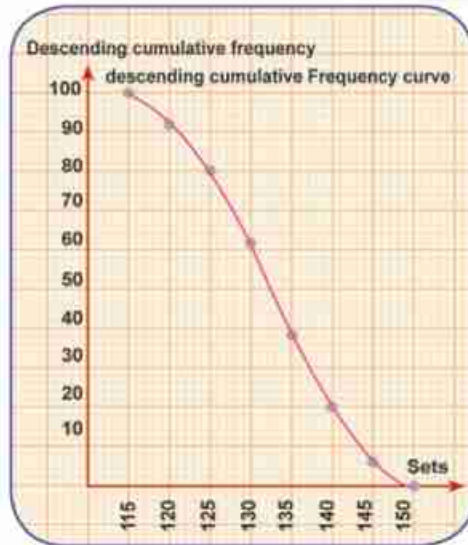
complete: 19 + ... + ... + ... + ... = ...

To answer these questions in an easier way, form the descending cumulative frequency table as follows :

Descending cumulative frequency table

Lower limits of sets	Ascending cumulative frequency
115 and more	100
120 and more	92
125 and more	80
130 and more	61
135 and more	38
140 and more	20
145 and more	7
150 and more	zero

Lower limits of sets	descending cumulative frequency
115 and more	92 + 8 = 100
120 and more	80 + 12 = 92
125 and more	61 + 19 = 80
130 and more	38 + 23 = 61
135 and more	20 + 18 = 38
140 and more	7 + 13 = 20
145 and more	0 + 7 = 7
150 and more	0

To represent this table graphically, follow the steps of representing the ascending cumulative frequency to get the following graphical representation:

Exercises (3 – 2)

1 **Below are the scores of 100 students in an experimental Maths exam.**

Sets	0–	10–	20–	30–	40–	50–	Total
Frequency	8	14	15	28	23	12	100

Required:

- **A** Form both the ascending and descending cumulative frequency tables.
- **B** Graph both the ascending and the descending cumulative frequency curves on the same graph paper.
- **C** From the graph, find the number of students who got less then 40 marks and those who got 40 marks and more.
- **D** Find the percentage of success, given that the minimum mark of success is 20 marks.
- **E** What is the percentage of the students who got more than 40 marks?

2 **The following table shows the frequency distribution of the scores of 50 students in an experimental math exam.**

Intervalle	2–	6–	10–	14–	18–	22–	26–	Total
Effectif	3	5	9	10	12	7	4	50

Required: **Graph the ascending cumulative frequency curve.**

3. **The following table shows the frequency distribution of the daily wages of some workers.**

Sets	5–	10–	15–	20–	25–	30–	Total
Frequency	10	14	24	30	12	10	100

Required: Graph the descending cumulative frequency curves.

4. **The following table shows the frequency distribution of the ages of 50 workers.**

Sets	20–	25–	30–	35–	40–	45–	50–	Total
Frequency	5	8	9	13	....	5	3	50

Required:

A. Complete the missing space.

B. Graph the ascending and descending cumulative frequency curves.

C. Find: First: the number of workers whose ages are more than 32.

Second: The number of workers whose ages are less than 43.

5. **The following table shows the frequency distribution of the scores of 1000 students in a final year exam.**

Percentages	20–	30–	40–	50–	60–	70–	80–	90–	Total
Number of students	30	70	160	260	150	130	110	90	1000

Required:

A. Graph the ascending and descending cumulative frequency curves.

B. Find the number of students whose scores are less than 75 marks.

C. Find the number of students whose scores are more than 85 marks.

Arithmetic Mean, Median and Mode

Think and Discuss

First: the mean

You have learned to find the mean for a set of values and learned that:

$$\text{The arithmetic mean} = \frac{\text{The sum of values}}{\text{Number of values}}$$

Example: If the ages of 5 students are 13، 15، 16، 14، and 17 years old, then

The mean of their ages $= \dfrac{13 + 15 + 16 + 14 + 17}{5}$

$= \dfrac{75}{5} = 15$ years

Remark: $15 \times 5 = 13 + 15 + 16 + 14 + 17$

The mean: is the simplest and most commonly used type of averages, It's that value given to each item in a set, then the total of these new values is the same total of the original values. It can be calculated by adding up all values, then divide the sum by the number of values.

Finding the mean of data from the frequency table with sets:

How can you find the mean of the following frequency distribution:

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	10	20	25	30	15	100

Remark: To find the mean for a frequency distribution with sets, follow the following steps:

You will learn how

- To find the mean from a frequency table with sets.
- To calculate the median from a frequency table.
- To calculate the mode from a frequency table with sets.

key terms

- Mean.
- Median.
- Frequency histogram.
- Mode.

1 **Determine the centers of sets:**

The center of the first set = $\frac{20+10}{2}$ = 15 . The center of the second set = $\frac{30+20}{2}$ = 25 ... and so on

Since the ranges of the subsets are equal and each = 10

We consider the upper limit of the last set = 60 and then :

$$\text{its center} = \frac{50+60}{2} = 55$$

2 **Form the following vertical table:**

Sets	Centre of the sets (X)	Frequency	Centre of the sets × frequency X × F
10 –	15	10	150
20 –	25	20	500
30 –	35	25	875
40 –	45	30	1350
50 –	55	15	825
Total		**100**	**3700**

3 **The mean =** $\frac{\text{The total of } (F \times X)}{\text{the total of } F}$

$$= \frac{3700}{100} = 37$$

Practice

1. If the mean of the scores of a student during the first 5 months is 23.8. What is the score of the 6th month If the mean of his scores is 24 marks?

2. The following table shows the frequency distribution of the weights of 30 children in kg.

Weight in (kg)	6–	10–	14–	18–	22–	26–	30–	Total
frequency	2	3	....	8	6	4	2	30

Complete the table, then find the mean of such a distribution.

Second: the median

The median is the middle value in a set of values after arranging it ascendingly or descendingy such that the number of values which are less than it is equal to the number of values which are greater than it.

Finding the median of a frequency distribution with sets graphically:

1. Draw the ascending or descending cumulative frequency table, then draw the cumulative frequency curve of it .
2. **Determine the order of the median =** $\frac{\text{The total of frequency}}{2}$.
3. Determine point A on the vertical axis (frequency) which represents the order of the median.
4. Draw a horizontal straight line from point A to intersect the curve at a point. Form this point, draw a vertical straight line on the horizontal axis to intersect it at a point that represants the median.

Example (1)

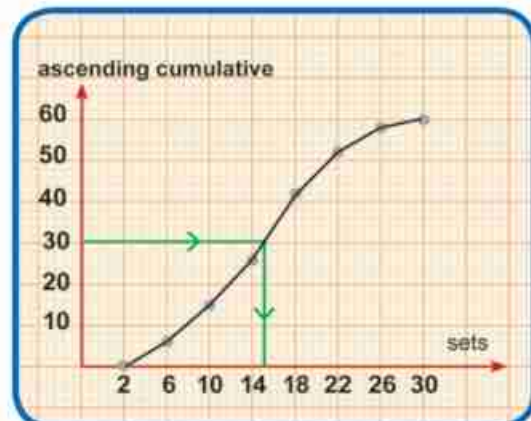
The following table shows the frequency distribution for the scores of 60 students in an exam.

Sets	2–	6–	10–	14–	18–	22–	26–	Total
Frequency	6	9	12	15	10	5	3	60

Find the median of the distribution using the ascending cumulative frequency table.

Solution

1. Draw an ascending cumulative frequency table.
2. Find the order of the median = $\frac{60}{2} = 30$
3. Draw the ascending cumulative frequency curve, and get the median form the graph.

The upper limits of the sets	The ascending cumulative frequency
Less than 2	0
Less than 6	6
Less than 10	15
Less than 14	27
Less than 18	42
Less than 22	52
Less than 26	57
Less than 30	60

From the graph, the median = 14.8 mark

Think up Can you find the median using the descending cumulative frequency table? Is the value of the median different in such a case?.

Example (2)

The following table shows the daily wages of 100 workers in a factory..

daily wages in LE (sets)	15-	20-	25-	30-	35-	40-	Total
Number of workers (frequency)	10	15	22	25	20	8	100

Required:

1. Graph the ascending and descending cumulative frequency curves on one figure.
2. Can you find the median wage from this curve?

Solution

Upper boundaries of sets	Cumulative frequency
Less than 15	zero
Less than 20	10
Less than 25	25
Less than 30	47
Less than 35	72
Less than 40	92
Less than 45	100

Lower boundaries of sets	Cumulative frequency
15 and more	100
15 and more	90
15 and more	75
15 and more	53
15 and more	28
15 and more	8
15 and more	zero

Remark:

The ascending cumulative frequency curve intersects with the descending cumulative frequency curve at one point which is m .

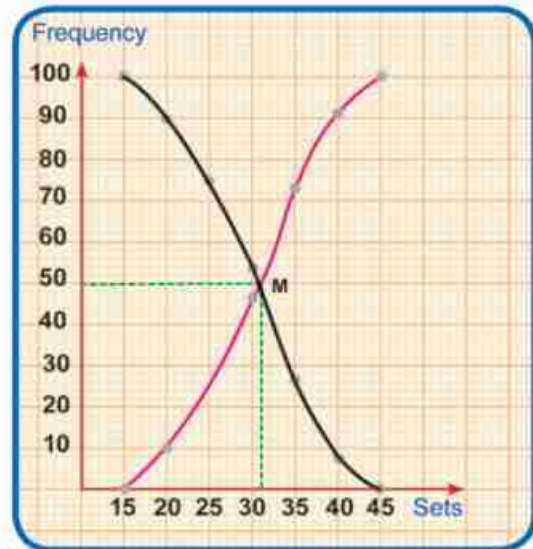
The y-coordinate for the point M = 50 $= \frac{100}{2}$

= the order of the median

∴ The X-coordinate of the point M determines the median

every 10 mm of the x coordinate represents L.E 5

Complete: 2 mm represents ....

The median wage = 30 + $\frac{2 \times 5}{10}$ = LE 31 .

Draw the descending cumulative frequency curve for the following frequency distribution, then find the value of the median.

Sets	5 –	10 –	15 –	20 –	25 –	30 –	total
Frequency	4	6	10	17	10	3	50

Third: the mode

The mode is the most common value in the set or in other words, it is the value which is repeated more than any other values.

The following table shows the frequency distribution for the scores of 40 students in an examination.

Sets	2–	6–	10–	14–	18–	22–	26–
Frequency	3	5	8	10	7	5	2

Find the mode of this distribution graphically

Solution

You can find the mode of this distribution graphically using the histogram as follows:

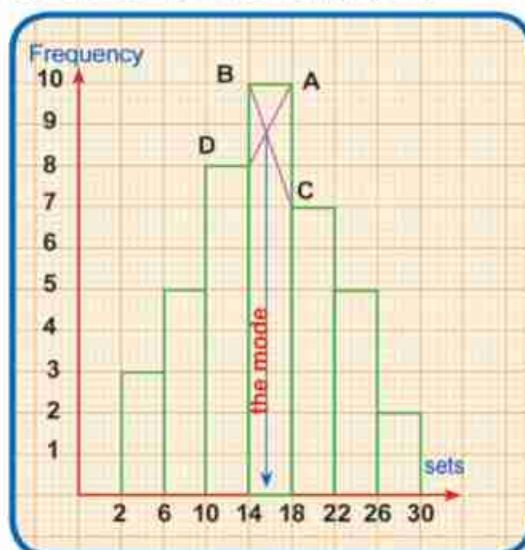
First: draw a histogram.

1. Draw two perpendicular axes: one horizontal to represent sets and the other vertical to represent the frequency of each set.

2. Divide the horizontal axis into a number of equal parts using a suitable drawing scale to represent sets.
3. Divide the vertical axis into a number of equal parts using a suitable drawing scale such that the greatest frequency among sets can be represented..
4. Draw a rectangle whose base is set (2-) and height is equal to the frequency (3).
5. Draw another rectangle adjacent to the first one whose base is set (6-) and height is equal to the frequency (5).
6. Repeat drawing the rest of adjacent rectangles till the last set (26-).

Second: Finding the mode from the histogram, to find the mode from the histogram, we observe that: the most repeated set is (14-), and it is called the mode set, why?

Define the intersection point of $\overline{AD}$, $\overline{BC}$ from the graph, and from this point, drop a vertical line on the horizontal axis to define the sequential value within that distribution.

From the graph, what's the mode value?

Exercises (3 – 3)

1 **The following table shows the frequency distribution of 50 workers days-off:**

Sets	2–	6–	10–	14–	18–	22–	26–
Frequency	4	5	8	K-2	7	5	1

 Find

A. the value of K
B. The Arithmetic mean for that distribution.

2 **The following table shows the frequency distribution of the heights of 120 students in centimeters:**

Height in (cm)	140–	144–	148–	152–	156–	160–	total
Frequency	12	20	38	22	17	11	120

Find the mean.

3 **The following table shows the frequency distribution of 50 workers' wages in a factory:**

sets of wages	300–	400–	500–	600–	700–	total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve, then find the median.

4 **From the following frequency table with equal sets in range.**

Sets	10–	20–	30–	40–	x –	60–	total
Frequency	12	15	25	27	k + 4	4	100

A. The value of both X and K.

B. Graph the ascending and descending cumulative Frequency curves in one figure, then find the median.

5 **The following table shows the frequency distribution of the weights of 50 students in Kg.**

weight in kg	30–	35–	40–	45–	50–	55–	total
Number of students	k + 4	3k	4k	3k + 1	3k - 1	k + 1	50

Find

A. the value of K.

B. Graph the frequency histogram, then find the mode weight.

6 **The following table shows the frequency distribution of the height of 200 students.**

Height in (cm)	110–	115–	120–	125–	130–	135–	140–	total
Number of students	10	12	28	35	60	40	40	200

Graph the frequency histogram, then find the mode length.

General Exercises

1 **The following table shows the frequency distribution for the scores of 50 students in an examination:**

Sets	2 –	6 –	10 –	14–	18–	22–	26–	total
Frequency	3	5	9	10	12	7	4	50

Find **First:** the mean of the student's score. **Second:** The median

2 **From the following frequency table with equal sets in range, find:**

Sets	10–	20–	x–	40–	50–	60–	total
Frequency	10	17	20	32	K + 2	4	100

First: find the value of X and K:

Second: graph the ascending and descending cumulative curves on one figure, then calculate the median

3 **Find** **the mode of the following frequency distribution for the scores of 40 student in an examination:**

Sets of marks	30–	40–	50–	60–	70–	80–	total
frequency	3	4	12	8	7	6	40

4 **The following table shows the frequency distribution with equal - range sets for the weekly wages of 100 works in a factory.**

Sets of wages in L.E	70–	80–	90–	100–	x–	120–	130–
Number of workers	10	13	f - 4	20	16	14	11

Trouver

A. The value of x and F

B. The mode of wages in L.E

Activity

The following table shows the frequency distribution for the weights of 50 students in K.g at a school.

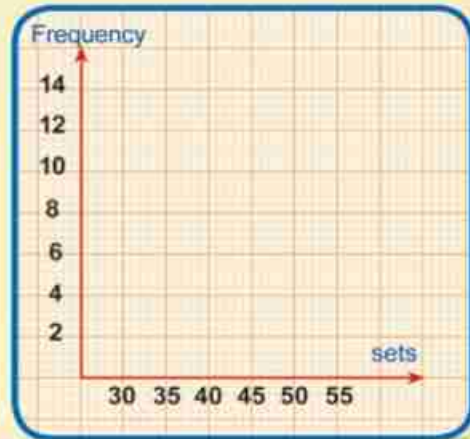
Weight in K.g	30 –	35 –	40 –	45 –	50 –	55 –	total
Number. of students	7	3 k	4k	10	8	4	50

First: find the value of K.

Second: calculate the mean.

Third: Draw the ascending cumulative frequency curve.

Fourth: Draw the histogram and find the mode of weights.

Fifth: Find the median.

Unit test

1 **Complete the following:**

- **A** If the lower limit of a set is 8 and the upper limit of the same set is 14, then its centre is ....
- **B** If the lower limit of a set is 4 and its center is 9, then its upper limit is ....
- **C** The intersection point of the ascending and descending cumulative frequency curves determines .... on the sets axis
- **D** The location of the top of the frequency curve on the set axis is ....
- **E** If the mean of a frequency distribution is 39.4 and the total of its frequency is 100, then the total of the product of multiplying each set frequency by its centre = ...

2 **The following table shows the frequency distribution of weights of 20 children in k.g**

sets	5 –	15 –	25 –	35 –	45 –	total
Frequency	3	4	7	4	2	20

Find the median weight in k.g. using the ascending and descending cumulative frequency curve of this distribution.

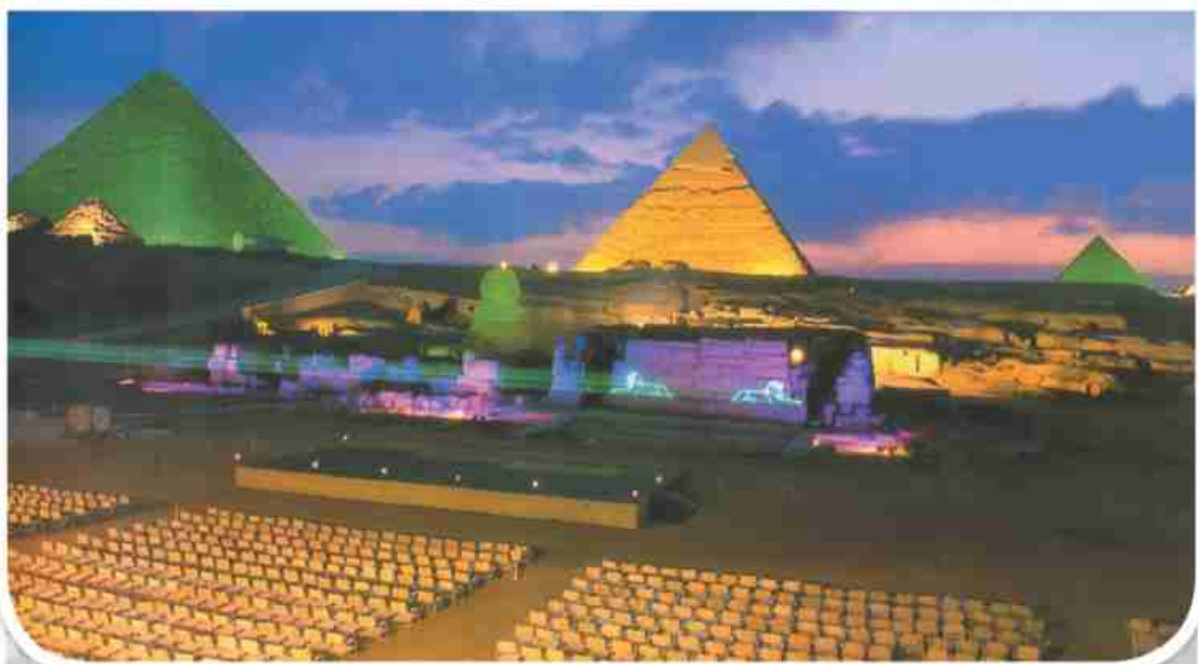
3 **below is the frequency distribution of the weekly bonus of 100 workers in a factory.**

Bonus in L.E.	20 –	30 –	40 –	50 –	60 –	70–
No. of workers	10	K	22	26	20	8

- **A** Calculate the value of K.
- **B** Find the mean of this distribution.
- **C** The mode value of the weekly bonus using the histogram..

UNIT FOUR

4

Geometry

Medians Of Triangle

Think and Discuss

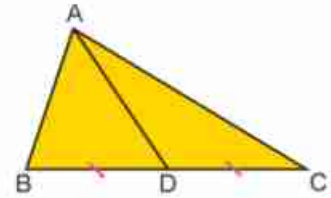
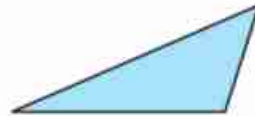
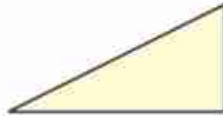
You will learn how

- Medians of the triangle.
- A 30° – 60° – 90° triangle.

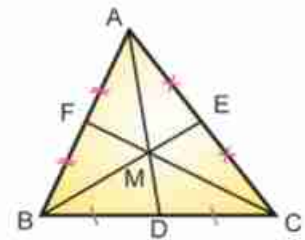
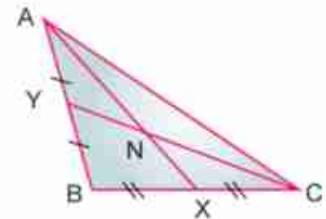
key terms

- Median of the triangle.
- A 30° – 60° – 90° triangle.
- Point of Concurrence

The medians of a triangle is the line segment drawn from the triangle vertex to the middle of the opposite side of this vertex.

ABC is a triangle where the point D bisects $\overline{BC}$.
So $\overline{AD}$ is a triangle Median.

- How many medians does the trianglle have?
- Draw the medians in each triangle.

Theorem 1

The medians of a triangle are concurrent

ABC is a triangle where point D bisects $\overline{BC}$, point E bisects $\overline{AC}$, point F bisects $\overline{AB}$, then $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ all intersect in one point (M)

In the Figure opposite: ABC is a triangle where point x bisects $\overline{BC}$, point y bisects $\overline{AB}$, and $\overline{AX} \cap \overline{CY} = \{N\}$.

1. Draw $\overrightarrow{BN}$ to intersect $\overline{AC}$ at point Z, then find the lengths of $\overline{AZ}$ and $\overline{CZ}$ is AZ = CZ? Reason your answer.

2. Measure, then complete.

$\frac{NX}{NA} = \frac{\ldots\ldots}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots}$, $\frac{NY}{NC} = \frac{\ldots\ldots}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots} = \frac{NZ}{NB} = \frac{\ldots\ldots}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots}$

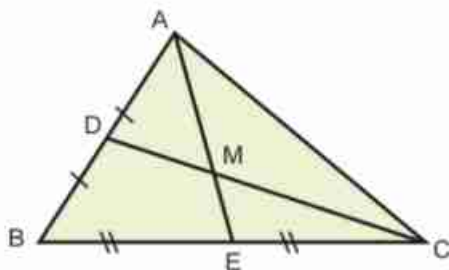
- If your measurements are accurate, then $\frac{NX}{NA} = \frac{1}{2}$, $\frac{NY}{NC} = \frac{1}{2}$ and $\frac{NZ}{NB} = \frac{1}{2}$

Theorem 2

The point of concurrence of the medians of the traingle divides each median in the ratio of 1:2 from its base

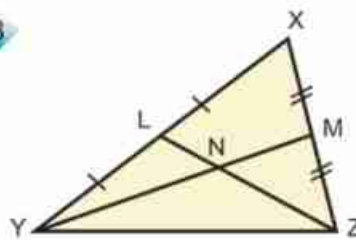
Practice

Complete

A

ME = 3cm, MC = 8cm

MA = ..., MD = ...

ME = ... AE, MC = ... CD

B

LZ = 15cm, YM = 18cm, XY = 20cm

NL = ..., NY = ...,

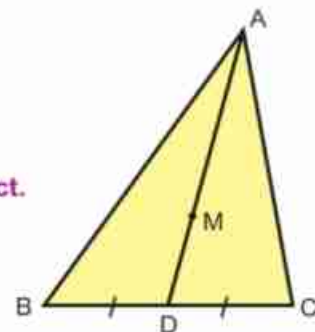
Perimeter of Δ NLY= ....,

Fact

$\overline{AD}$ is a median in Δ ABC, M $\in$ $\overline{AD}$

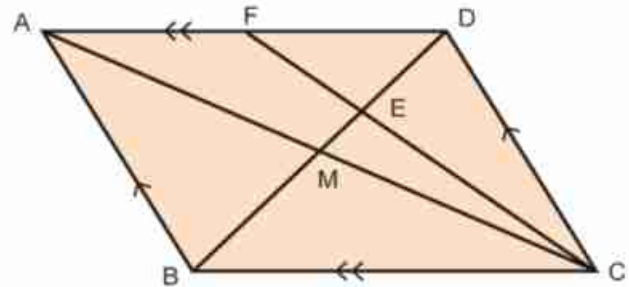
if AM = 2 MD,

then

then **M is the point where the medians of the triangle intersect.**

Example (1)

In the figure opposite: ABCD is a parallelogram where its two diagonals intersect at point M, point $E \in \overline{DM}$ and DE = 2 EM.

$\overrightarrow{CE}$ is drawn and intersected $\overline{AD}$ at point F.

Prove that: AF = FD

Proof: In the parallelogram ABCD

$\because \overline{AC} \cap \overline{BD} = \{M\}$

$\therefore$ M bisects $\overline{AC}$

the triangle DAC

$\because$ M bisects $\overline{AC}$

$\therefore \overline{DM}$ is a median of the triangle.

$\because E \in \overline{DM}$, DE = 2 EM.

$\therefore$ E is the intersecting point of the triangle's medians.

$\because E \in \overline{CF}$

$\therefore \overline{CF}$ is a median of the triangle and point F bisects $\overline{AD}$

Theorem 3

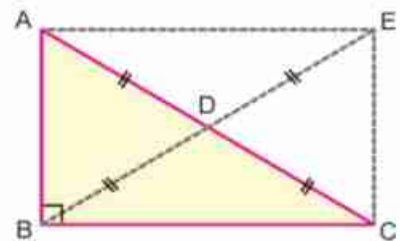
In the right - angled triangle, the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

Given data: ABC is a triangle where m ($\angle$ B) = 90°, $\overline{BD}$ is a median in $\triangle$ ABC.

Required : Prove that: BD = $\frac{1}{2}$ AC.

Construction : Draw $\overrightarrow{BD}$, let point $E \in \overrightarrow{BD}$ where BD = DE.

Proof :

$\because$ in the Figure ABCE, $\overline{AC}$, $\overline{BE}$ bisect each other.

∴ the Figure ABCE is a parallelogram.

∵ m (∠ B) = 90°

∴ ABCE is a rectangle.

∴ BE = AC.

∵ BD = $\frac{1}{2}$ BE ∴ BD = $\frac{1}{2}$ AC Q.E.D.

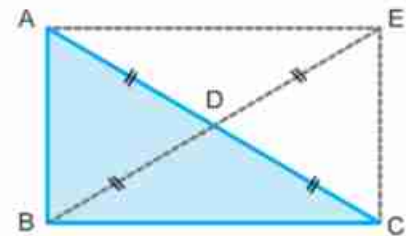
The converse of the theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

Given: ABC is a triangle where $\overline{BD}$ is a median,

BD = DA = DC

Required: Prove that: m (∠ ABC) = 90°.

Construction : Draw $\overrightarrow{BD}$ and let point E ∈ $\overrightarrow{BD}$ where BD = DE.

Proof: ∵ BD = $\frac{1}{2}$ BE = $\frac{1}{2}$ AC.

∴ BE = AC.

In the Figure ABCE, AC and BE are equal in length and bisect each other.

∴ the Figure ABCE is a rectangle.

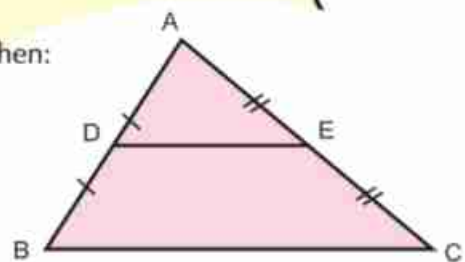
∴ m (∠ ABC) = 90°) Q.E.D.

Corollary

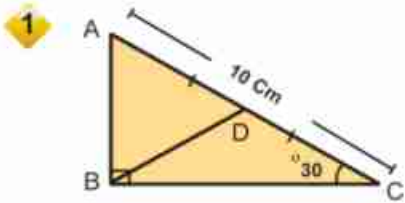
The length of the side opposite to the angle of measure 30° in the right - angled triangle equals half the length of the hypotenuse.

Remember:

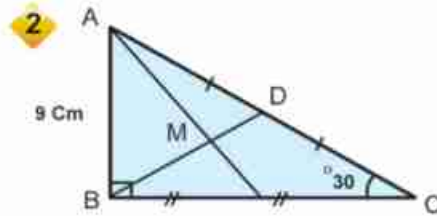
If the point D bisects $\overline{AB}$ and the point E bisects $\overline{AC}$, Then:

1. DE = $\frac{1}{2}$ BC
2. $\overline{DE}$ // $\overline{BC}$

Exercises (4-1)

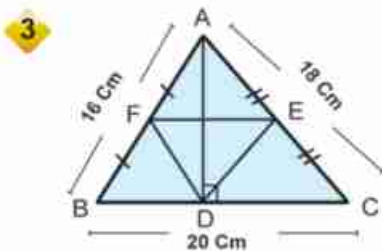
Complete:

1

BD = ... cm, AB = ... cm

Perimeter △ ABD = ... cm

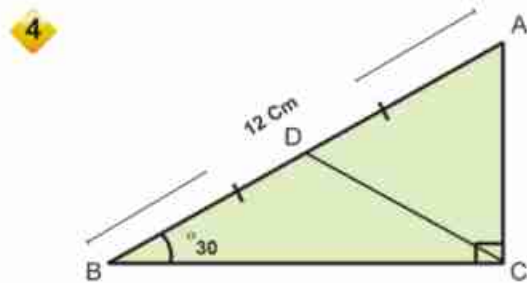
2

AC = ... cm, BD = ... cm

MD = ... BD, MD = ... cm

3

DF = ... cm, DE = ... cm,

FE = ... cm

Perimeter △ DEF = ... cm

4

AC = ... cm, AD = ... cm

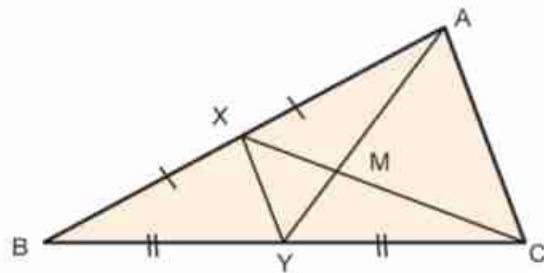
BC = ... cm, CD = ... cm

5 **In the figure opposite:**

ABC is a triangle, X bisect $\overline{AB}$,

Y bisect $\overline{BC}$, XY = 5 cm,

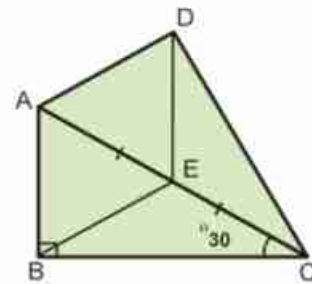
$\overline{XC} \cap \overline{AY}$ = {M}

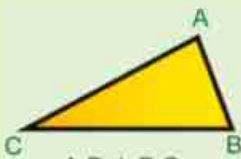
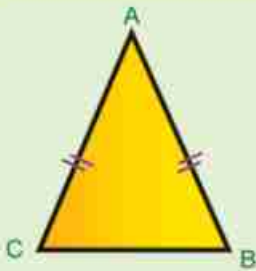
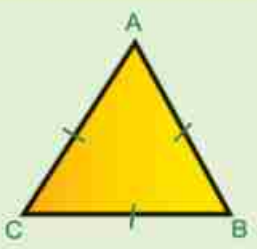
where: CM = 8cm, YM = 3 cm

Find:

(1) The perimeter of △ MXY

(2) The perimeter of △ MAC

6 ABC is a triangle where point D bisects $\overline{BC}$, and point M ∈ $\overline{AD}$. AM = 2 MD.
Draw $\overrightarrow{CM}$ to intersect $\overline{AB}$ at point E.
If EC = 12 cm,
then find the length of $\overline{EM}$

7 **In figure opposite:**
ABC is a right-angled triangle at B,
m (∠ ACB) = 30°.
AB = 5 cm, point E is the mid point of $\overline{AC}$.
If DE = 5 cm, then prove that m (∠ ADC) = 90°.

The Isosceles Triangle

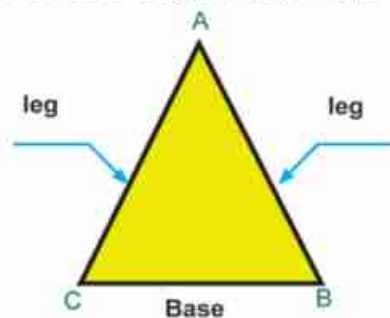
Think and Discuss

You will learn how
- To define the properties of the isosceles triangle.
- To define the classifications of the isosceles triangle..

key terms
- The isosceles triangle.
- The equilateral triangle.
- The scalene triangle.

You have learnt that triangles are classified according to the lengths of their sides into three types:

The scalene triangle	The isosceles triangle (two sides are congruent)	The equilateral triangle (three sides are congruent)
A, B, C $AB \neq BC$ $AB \neq AC$ $BC \neq AC$	A, B, C $AB = AC$	A, B, C $AB = AC = BC$

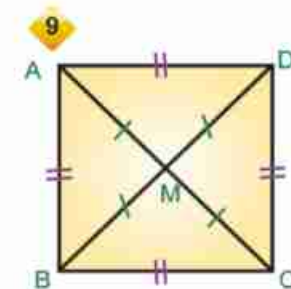
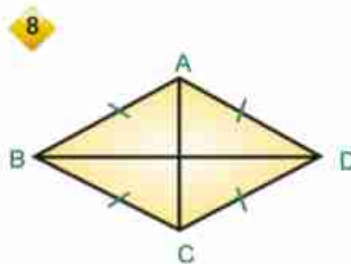
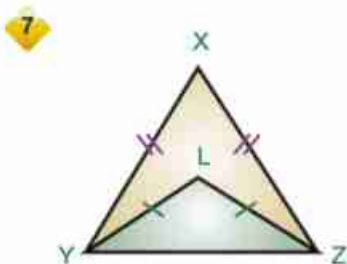
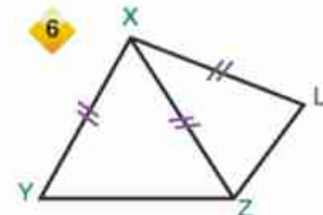
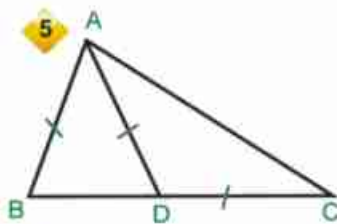
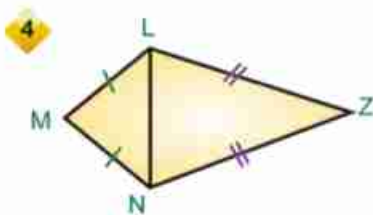
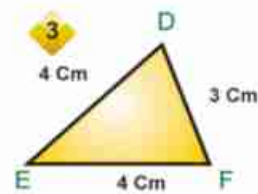
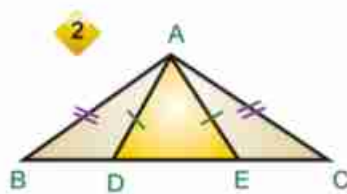
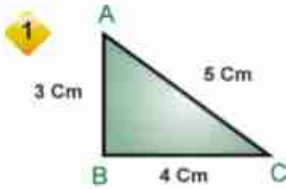
In the figure opposite :

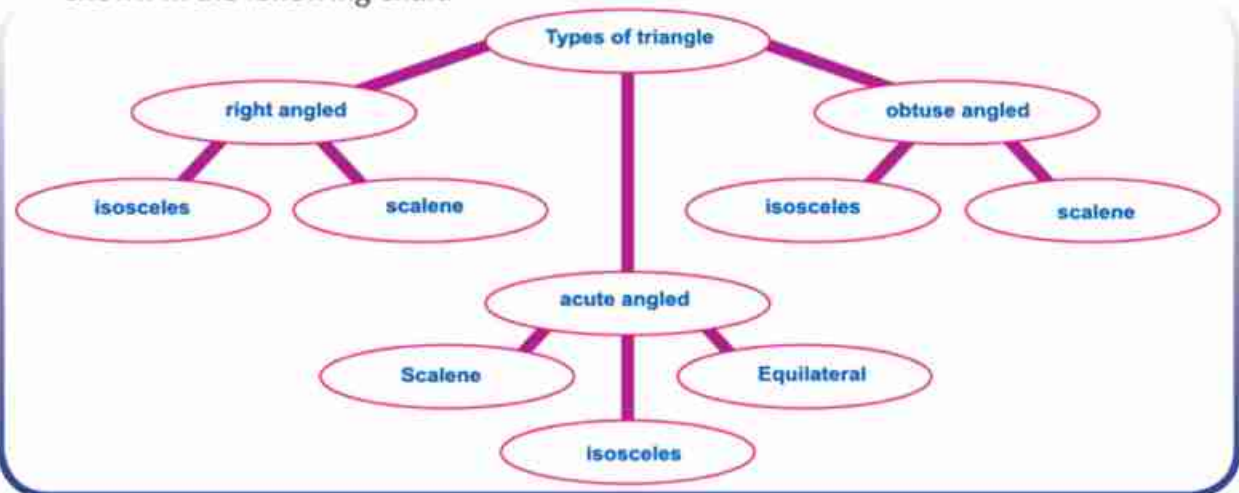
Remark : the two sides $\overline{AB}$, $\overline{AC}$ are congruent (of equal lengths), so the triangle ABC is called isosceles triangle while the point A is called the vertex. $\overline{BC}$ is the base, and the two angles B and C are the base angles of the triangle.

The properties of isosceles triangle

In any isosceles triangle

- What is the type of the base angles? (acute - right - obtuse)
- What is the type of the vertex angle?

Practice

In each of the following figures, state the isosceles triangles and define their bases, then notice the type of the two base angles and the vertex angle.

Remark :

1. Both of the base angles in the isosceles triangle are acute.
2. The vertex angle in the isosceles triangle can be either acute, right or obtuse. So, the isosceles triangle can be either obtuse, right or acute angled triangle as shown in the following chart:

The Isosceles Triangle Theorems

Think and Discuss

You will learn how

- To define the relation between the base angles in the isosceles triangle.
- To define the relation among the measures of the angles in the equilateral triangle.
- To define the relation between two sides opposite to two equal angles in a triangle.
- To know that if the angles in a triangle are congruent, then the triangle is equilateral.

key terms

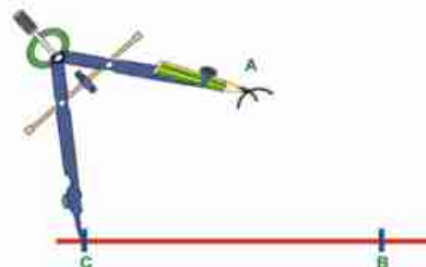
- The isosceles triangle.
- The base angles.

Is there a relation among the measures of the two base angles in the isosceles triangle?

to know that, let's conduct the next activity:

Activity

Using the compass

1. Draw several isosceles triangles as shown in the opposite figure Where AB = AC.
2. **Find** using a protractor, the measure of the two base angles ∠ ABC and ∠ ACB
3. Write down the data you got in a table as follows, then compare the measures in each case.

Number of the triangle	m (∠ ABC)	m (∠ ACB)
1		
2		
3		

4. Keep your activity in the portfolio.

Theorem 1

(the isosceles triangle theorem) the base angles of the isosceles triangle are congruent.

Given: ABC is triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P: ∠ B ≡ ∠ C

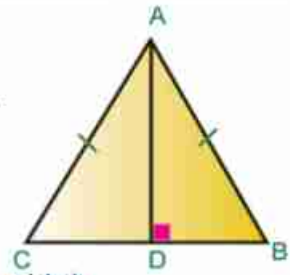
Construction : draw $\overline{AD} \perp \overline{BC}$

Proof : The two triangles ADB and ADC are right angled in which.

$$\begin{cases} \overline{AB} \equiv \overline{AC} & \text{(a given)} \\ \overline{AD} & \text{(a common side)} \end{cases}$$

$\therefore \Delta\, ADB \equiv \Delta\, ADC$ (a hypotenuse and a side)

from the congruency , we deduce that

$\angle B \equiv \angle C$ Q.E.D.

practice

1 **In each of the following figures, find the value of the symbol that is used to measure the angle:**

A

x = ...

B

y = ...

C

N = ... E = ...

D

x = ... , y = ... , l = ...

E

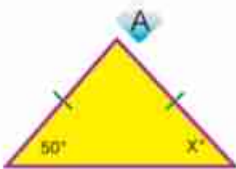
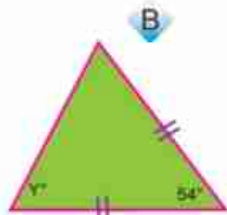
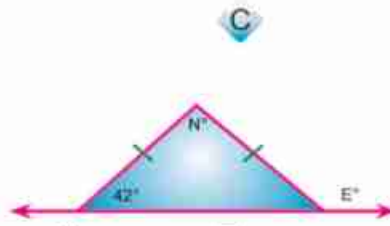
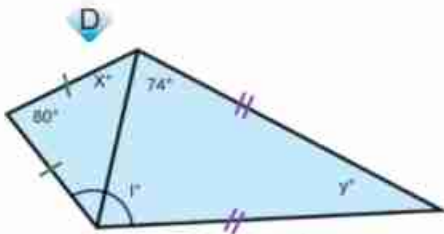
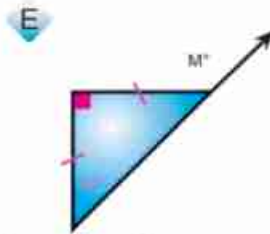
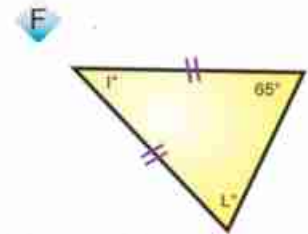
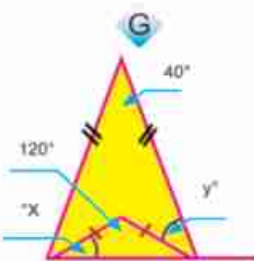
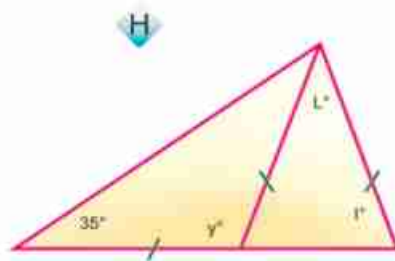
L = ... , M = ...

F

L = ... , l = ...

G

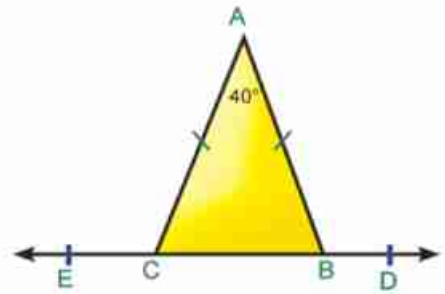
x = ... , y = ...

H

y = ... , L = ... , l = ...

2 **In the figure opposite,** ABC is an isosceles triangle in which AB = AC

$m(\angle A) = 40°$, $D \in \overrightarrow{CB}$, $E \in \overrightarrow{BC}$.

First: **Find** $m(\angle ABC)$

Second: **Prove that** $\angle ABD \equiv \angle ACE$

Think: Are the supplementary angles to congruent angles congruent?

Corollary

If the triangle is equilateral, then it is equiangular where each angle measure 60°.

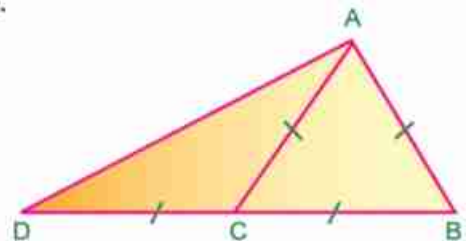
Example (1)

In the figure opposite: ABC is an equilateral triangle.

$D \in \overrightarrow{BC}$ such that BC = CD

prove that $\overline{AB} = \overline{AD}$

Given: $AB = BC = CA = CD$, $D \in \overrightarrow{BC}$

R.T.P: Prove that $\overline{BA} = \overline{AD}$

Proof : $\because$ Δ ABC is an equilateral triangle.

$\therefore m(\angle ACB) = m(\angle BAC) = m(\angle B) = 60°$ **(corollary)**

$\because D \in \overrightarrow{BC}$

$\therefore \angle BCA$ is an exterior angle of the $\triangle ACD$

$m(\angle BCA) = m(\angle CAD) + m(\angle CDA) = 60°$ **(1)**

In $\triangle ACD$

$\because CA = CD$ $\therefore m(\angle CAD) = m(\angle CDA)$ **(2)**

from (1), (2) we deduce that: $m(\angle CAD) = m(\angle CDA) = 30°$

$\because$ m ($\angle$ BAD) = m ($\angle$ BAC) + m ($\angle$ CAD)

$\therefore$ m ($\angle$ BAD) = 60° + 30° = 90°

$\therefore \overline{BA} \perp \overline{AD}$ Q.E.D

Remark:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non - adjacent interior angles.

Example (2)

2 **In the figure opposite:** AB = AD, BC = CD

Prove that $\angle$ ABC ≡ $\angle$ ADC

Given: AB = AD, BC = CD

R.T.P: **prove that** $\angle$ ABC ≡ $\angle$ ADC

Proof : In △ ABD

$\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB) **(1)**

in △ CBD

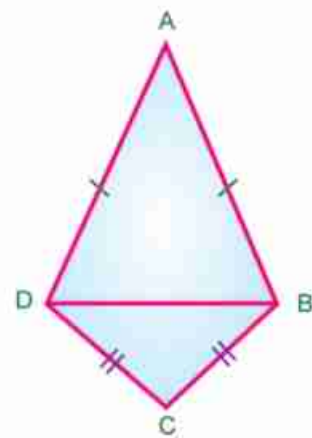
$\because$ CB = CD

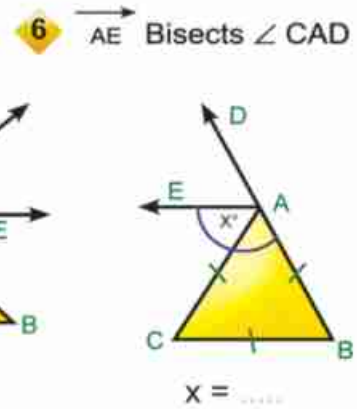
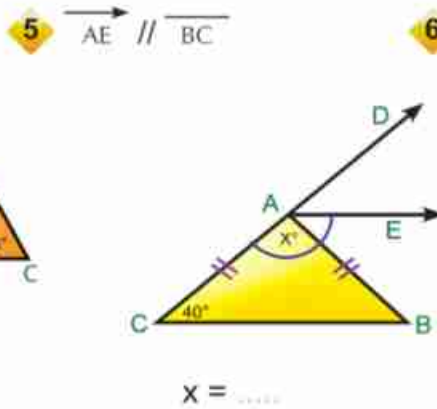
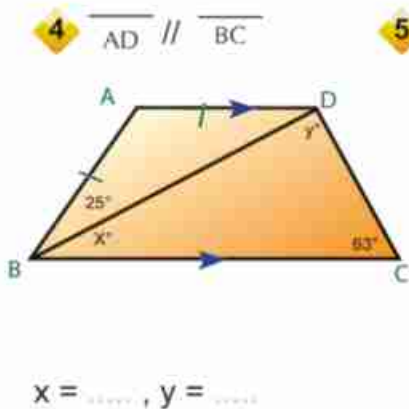
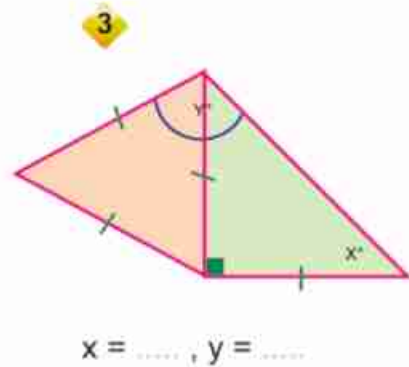
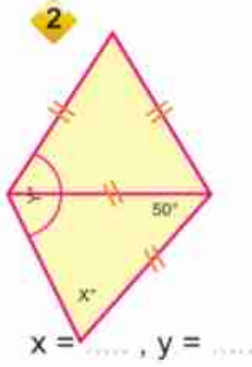
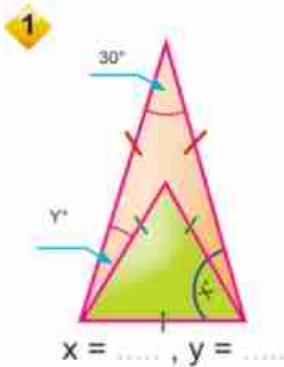
$\therefore$ m ($\angle$ CBD) = m ($\angle$ CDB) **(2)**

By adding (1) and (2) we deduce that

m ($\angle$ ABD) + m ($\angle$ CBD) = m ($\angle$ ADB) + m ($\angle$ CDB)

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ADC)

$\angle$ ABC ≡ $\angle$ ADC **Q.E.D.**

In each of the following figures, find the value of the symbol that is used to measure the angle:

x = ... , y = ... x = ... x = ...

Activity

Draw the triangle ABC in which BC = 7 cm, m (∠ B) = m (∠ C) = 50°, then measure the lengths of both $\overline{AB}$ and $\overline{AC}$. Repeat the activity using other measures for the length of $\overline{BC}$ and the measures of angles B and C, then fill in the table:

Number of the triangle	BC	m (∠ B)	m (∠ C)	AB	AC
1	7cm	50°	50°	......	......
2	......	......	......	......	......
3	......	......	......	......	......
4	......	......	......	......	......

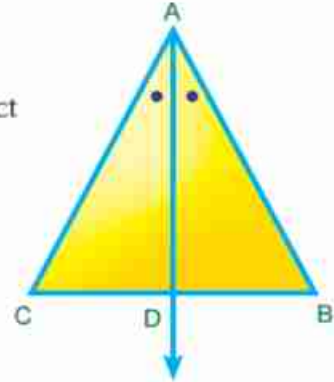
1. Are $\overline{AB}$ and $\overline{AC}$ equal in length?
2. Is $\overline{AB} \equiv \overline{AC}$?
3. How can you explain such corollaries geometricaly?

Theorem (2)

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.

ΔABC, $\angle B \equiv \angle C$

R.T.P: Prove that: $\overline{AB} \equiv \overline{AC}$

Construction : bisect $\angle BAC$ with the bisector $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D

Proof : $\because \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle CAD)$

$\because$ the sum of the measures of interior angles of a triangle is = 180 °

$\therefore m(\angle ADB) = m(\angle ADC)$

$\therefore$ In the two triangles ADB, ADC

$\overline{AD}$ is a common side
$m(\angle BAD) = m(\angle CAD)$
$m(\angle ADB) = m(\angle ADC)$

$\therefore \Delta ADB \equiv \Delta ADC$

Form the congruency, we deduce that $\overline{AB} \equiv \overline{AC}$

Therefore, ΔABC is an isosceles triangle

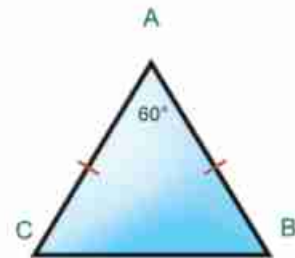
Corollary

If the angles of a triangle are congruent, then the triangle is equilateral.

In the figure opposite ABC is an isosceles triangle in which:
$AB = AC$, $m(\angle BAC) = 60°$

Complete $m(\angle ABC) = m(\angle ACB) = \dots\dots$

i.e: $\angle \dots\dots \equiv \angle \dots\dots \equiv \angle \dots\dots$

$\therefore \Delta ABC$ is .... triangle

Remark: In an isosceles triangle, If any angle has a measure of 60°, then the triangle is an equilateral triangle.

practice

In each of the following figures, define the triangle`s sides that are equal in length as shown in example 1 :

1.
AB = AC

2.
.... = ...

3.
.... = ...

4.
.... = ...

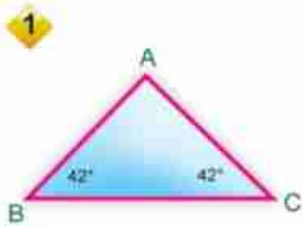
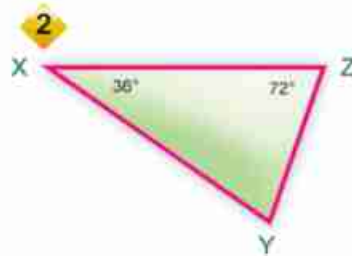
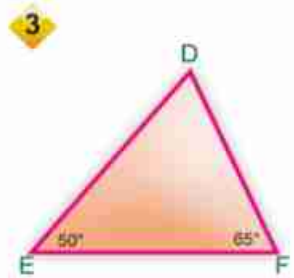
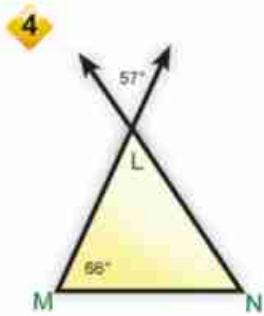
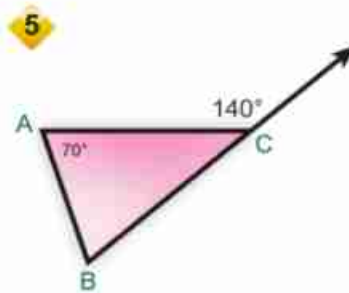
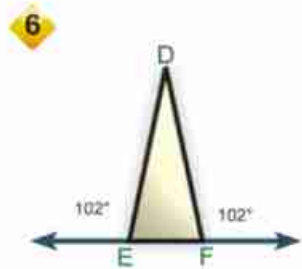
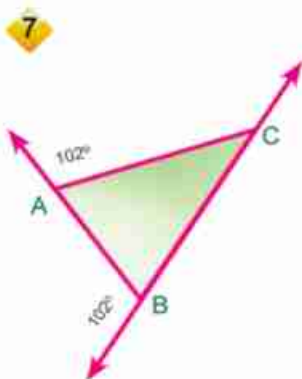
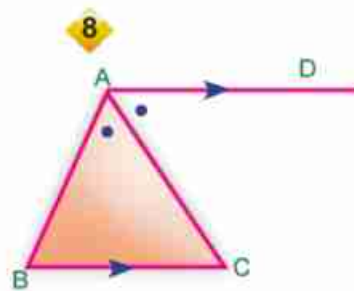
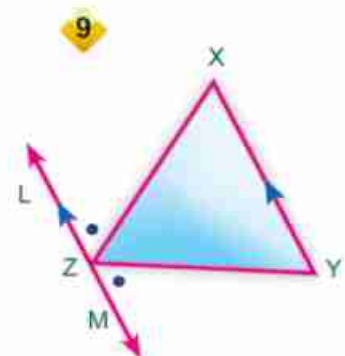
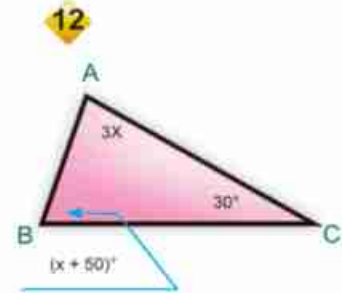
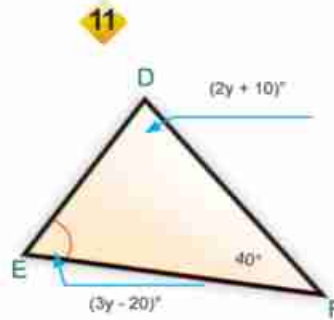
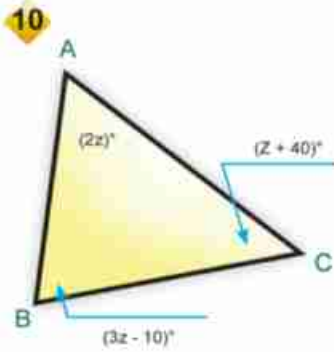
5.
.... = ...

6.
.... = ...

7.
.... = ...

8.
.... = ...

9.
.... = ...

10
11
12
.... = ... = ... = ...

Examples

1 **In the figure opposite:** ABC is a triangle in which AB = AC, $\overline{XY}$ // $\overline{BC}$

prove that Δ AXY is an isosceles triangle

Given: AB = CA , $\overline{XY}$ // $\overline{BC}$.

Required: prove that AX = AY

Proof: In △ ABC ∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) (1)

∵ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ a transversal

∴ m (∠ AXY) = m (∠ ABC) correspondingly (2)

The same $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ a transversal

∴ m (∠ AYX) = m (∠ ACB) correspondingly (3)

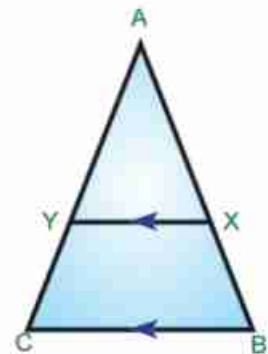
from (1), (2), (3) we deduce that :

m (∠ AXY) = m (∠ AYX)

In Δ AXY

∵ m (∠ AXY) = m (∠ AYX)

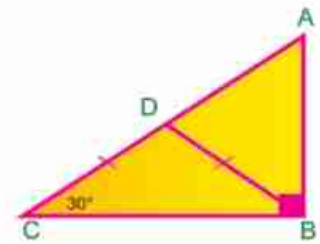
∴ AX = AY

i.e. the triangle AXY is an isosceles triangle Q.E.D

Think : Can we deduce that XB = YC ? Explain your answer,

2 **In the figure opposite :**

ABC is a right angled triangle at B, m (∠ C) = 30°,

D ∈ $\overline{AC}$ where DB = DC

 prove that Δ ABD is an equilateral triangle.

Given: m (∠ A B C) = 90° , m (∠ C) = 30° , D B = D C

R.T.P: prove that A B = B D = AD

proof: In △ D B C ∵ D B = D C

∴ m (∠ D B C) = m (∠ C) = 30°

in △ A B C ∵ m (∠ AB C) =90° , m (∠ D B C) = 30°

∴ m (∠ BAD) =90 - 30 = 60° **(1)**

∵ ∠ ADB is an exterior angle of △ B D C

∴ m (∠ A D B) = m (∠ DBC) + m (∠ DCB)

m (∠ ADB) = 30° + 30° = 60° **(2)**

In Δ ABD ∵ the sum of the measures of the interior angles of a triangle = 180°

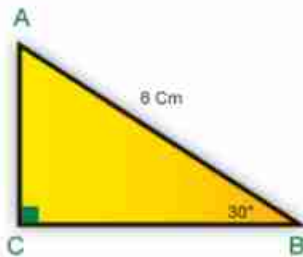
∴ m (∠ ABD) = 180° - (60° + 60°) = 60° **(3)**

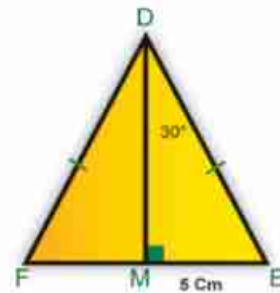
from (1), (2), (3) ∴ m (∠ ABD) = m (∠ ADB) = m (∠ A)

i.e. ∠ ABD ≡ ∠ ADB ≡ ∠ A

∴ the triangle ABD is equilateral i.e. AB = B D =AD

Exercises (4-3)

  Complete:

A C = ...

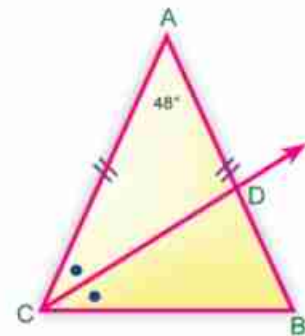
XZ = ... cm

DE = ... cm, m (∠E) = ...°

EF = ...cm, m (∠MDF) = ...°

2 **In the figure opposite :**

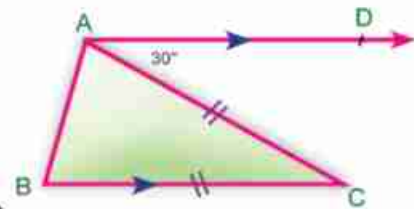
A B = A C , m (∠ BAC) = 48°

$\overrightarrow{CD}$ bisects ∠ B C A and intersects $\overline{AB}$ at D

 Find m (∠ B) et m (∠ BCD)

3 **In the figure opposite :**

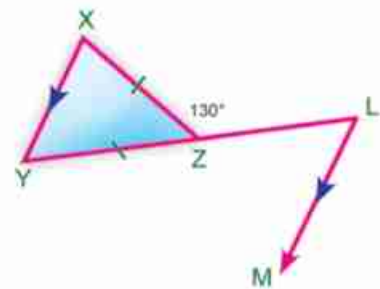
A B C is a triangle in which AC = BC،

$\overrightarrow{AD}$ // $\overline{BC}$, m (∠ D A C) = 30°

 Find The measures of the angles in △ ABC

4 **In the figure opposite :**

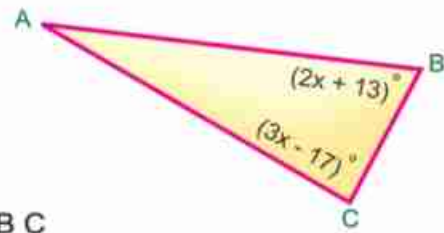
Z ∈ $\overline{LY}$, X Z = Y Z

m (∠ L Z X) = 130° , $\overrightarrow{LM}$ // $\overline{XY}$.

 Find m (∠ MLY)

5. **In the figure opposite :**

   $AB = AC$, $m(\angle B) = (2x + 13)^\circ$

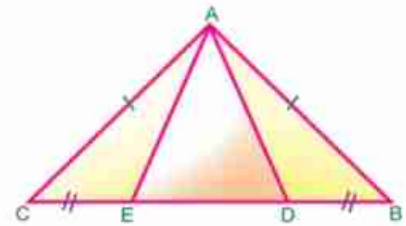
   $m(\angle C) = (3x - 17)^\circ$

Find the measures of the angles of ΔABC

6. **In the figure opposite:**

   ABC is an isosceles triangle in which $AB = AC$,

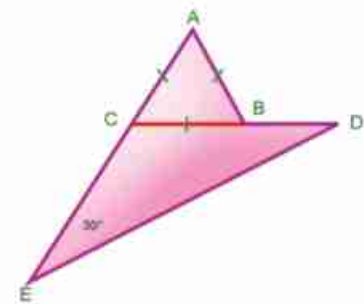
   $D \in \overline{BC}$, $E \in \overline{BC}$ such that $BD = EC$

Prove that **First:** ΔADE is an isosceles triangle.

Second: $\angle ADE \equiv \angle AED$

7. **In the figure opposite :** ABC is an equilateral traingle.

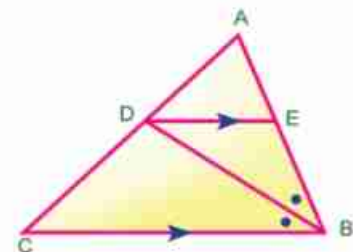
   $E \in \overrightarrow{AC}$, $D \in \overrightarrow{CB}$,

   $m(\angle DEC) = 30^\circ$

Prove that ΔDCE is an isosceles triangle.

8. **In the figure opposite :**

   $\overrightarrow{BD}$ bisects $\angle ABC$ and intersects $\overline{AC}$ at D,

   $\overline{DE} \parallel \overline{BC}$ where $E \in \overline{AB}$.

Prove that ΔEBD is an isosceles triangle.

9. ABC is a triangle in which $D \in \overline{AB}$,

   $E \in \overline{BC}$ such that $BD = BE$, So if $\overline{DE} \parallel \overline{AC}$

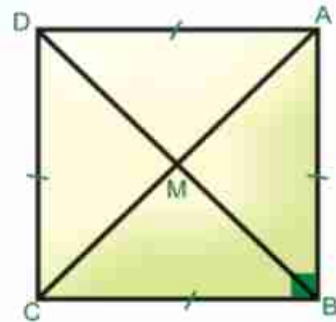
Prove that $AB = BC$

10. ABC is a triangle in which $AB = AC$,

    $\overrightarrow{BD}$ bisects $\angle ABC$, $\overrightarrow{CD}$ bisects $\angle ACB$, $\overrightarrow{BD} \cap \overrightarrow{CD} = \{D\}$

Prove that ΔDBC is an isosceles triangle.

ABCD is a square in which its digonals $\overline{AC}$, $\overline{BD}$ intersect at M

 Complete and discuss.

A. In ΔABC , m ($\angle ABC$) = ...°

$\because$ AB = B C

$\therefore$ m ($\angle BAC$) = m ($\angle BCA$) = ...°

B. $\because$ m ($\angle BAD$) = 90° $\therefore$ m ($\angle DAC$)= ....°

$\because$ m ($\angle BCD$) = 90° $\therefore$ m ($\angle ACD$)= ....°

C. Does the diagonal $\overline{AC}$ bisect $\angle$ A?

D. Does the diagonal $\overline{BD}$ bisect both $\angle$ B and $\angle$ D?

E. Is Δ M A D an isosceles triangle? why?

F. State those isosceles triangles whose vertices are point M.

G. Is M the midpoint of $\overline{AC}$ and $\overline{BD}$?

H. Is $\overline{AC} \equiv \overline{BD}$?

I. From the previous clauses, deduce the properties of the square and keep them in

Corollaries of isosceles triangle theorems

Think and Discuss

You will learn how

- The corollaries on the theorems of isosceles triangles.

key terms

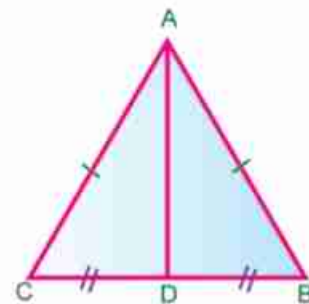
- The isosceles triangle
- The bisector of a vertex angle
- The bisector of a triangle base.
- The axis of symmetry for a line segment..

Corollary (1)

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the figure opposite:

In $\triangle ABC$, $AB = AC$, $\overline{AD}$ is a median

then: $\overrightarrow{AD}$ **bisects** $\angle BAC$. $\overline{AD} \perp \overline{BC}$

Remark: $\triangle ADB \equiv \triangle ADC$. Why?

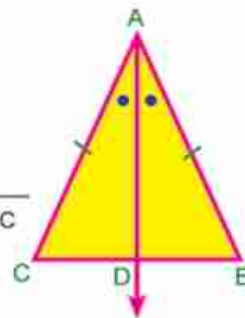
Corollary (2)

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the figure opposite:

In $\triangle ABC$, $AB = AC$,

$\overrightarrow{AD}$ bisects $\angle BAC$

then D is a midpoint of $\overline{BC}$ and $\overline{AD} \perp \overline{BC}$

Remark: $\triangle ADB \equiv \triangle ADC$. why?

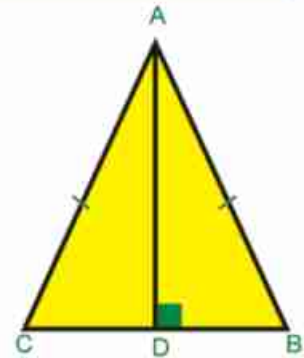
Corollary (3)

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

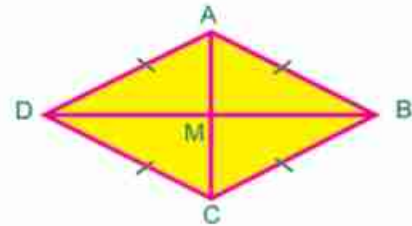
In the figure opposite :

In ΔABC, $AB = AC$, $\overline{AD} \perp \overline{BC}$

then D bisects $\overline{BC}$, $m(\angle BAD) = m(\angle CAD)$

Remark : $\Delta ADB \equiv \Delta ADC$. why?

Think

In the figure opposite :

ABCD is a quadrilateral in which all sides are equal in length, this figure is called rhombus, its diagonals are $\overline{AC}$ and $\overline{BD}$. they intersect at point M

Remark : $\Delta ABD \equiv \Delta CBD$. why?

$\therefore m(\angle ABD) = m(\angle CBD)$

in ΔABC, $AB = BC$, $\overrightarrow{BM}$ bisects $\angle ABC$

$\therefore \overline{BM} \perp$ .... , M is the midpoint of $\overline{AC}$

in ΔBAD, $AB = AD$, $\overline{AM} \perp \overline{BD}$

$\therefore \overrightarrow{AM}$ bisects $\angle$ .... , M is the midpoint of $\overline{BD}$

Are the two diagonals of the rhombus perpendicular?

Do the two diagonals of the rhombus bisect each other?

Does the diagonal of the rhombus bisect the vertex angles which it connects?

Write down your answer.

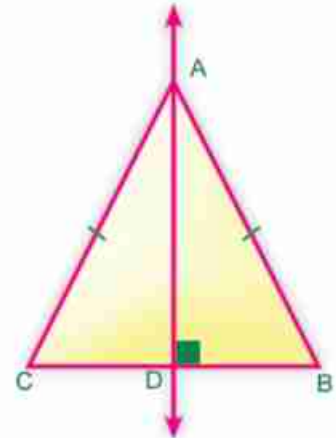
Axes of symmetry

First: axes of symmetry in the isosceles triangle:

The axis of symmetry of the isosceles triangle is the straight line drawn from the vertex angle perpendicular to its base.

In the figure opposite:

ΔABC in which $AB = AC$, $\overline{AD} \perp \overline{BC}$

then $\overleftrightarrow{AD}$ is the axis of symmetry in the isosceles triangle ABC.

Discuss:

Does the isosceles triangle has more than one axis of symmetry?

- How many axes of symmetry are there in the equilateral triangle?

- Are there any axes of symmetry in the scalene triangle?

Second: Axis of symmetry of a line segment :

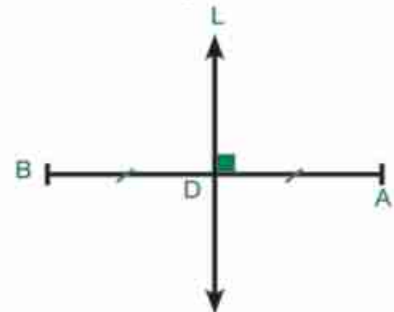
The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment in brief it is known as the axis of a line segment.

In the figure opposite:

If D the midpoint of $\overline{AB}$ and

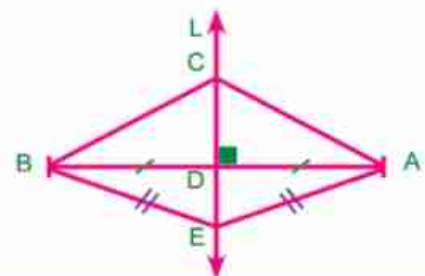
The straight line $L \perp \overline{AB}$

Where $D \in L$, then the straight line L

Is the axis of $\overline{AB}$

Important property

Any point at the axis of symmetry of a line segment is at equal distances from its end points.

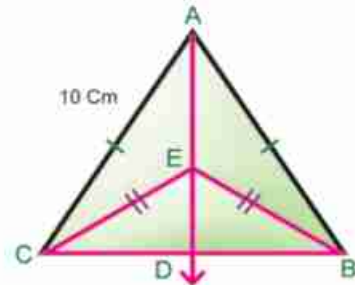
Remark:

1. If $C \in L$ then $AC = BC$
2. If $EA = EB$ then $E \in L$. why?

Examples

1 **In the figure opposite**

$AB = AC = 10$ cm , $EB = EC$

$\overrightarrow{AE} \cap \overline{BC} = \{D\}$

If $BC = 6$ cm, find the length of $\overline{CD}$ and $\overline{AD}$

Given : $AB = AC$, $EB = EC$

R.T.P : Find CD and A D

Proof :

$\because AB = AC$ $\quad \therefore$ A is on the axis of $\overline{BC}$

$\because EB = EC$ $\quad \therefore$ E is on the axis of $\overline{BC}$

$\therefore \overleftrightarrow{AE}$ is the axis of $\overline{BC}$

D is the midpoint of $\overline{BC}$, $\overline{AD} \perp \overline{BC}$

$\because$ D is the midpoint of $\overline{BC}$, $BC = 6$cm $\quad \therefore CD = 3$cm

$\because \overline{AD} \perp \overline{BC}$

$\therefore$ In Δ ADC that is right angled triangle at D

$(AD)^2 = (AC)^2 - (CD)^2$

$(AD)^2 = 100 - 9$

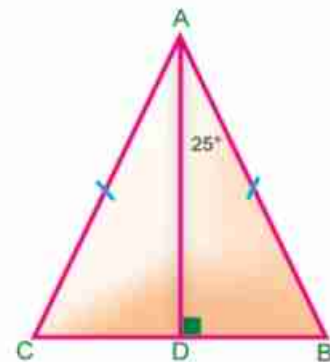
$\therefore AD = \sqrt{91}$ cm

2 **In the figure opposite**

A B C is a triangle in which $AB = AC$,

$\overline{AD} \perp \overline{BC}$, $m(\angle BAD) = 25°$,

$BC = 4$ cm. Find:

A $m(\angle DAC)$ **B** the length of $\overline{DC}$

Solution

Given: $AB = AC$,

$\overline{AD} \perp \overline{BC}$, $m(\angle BAD) = 25°$, $BC = 4$ cm

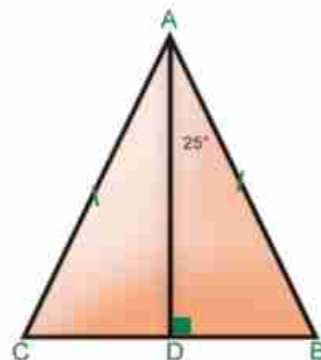
R.T.P: $m(\angle DAC)$, and the length of $\overline{DC}$.

Proof : in △ ABC

∵ A B = A C , $\overline{AD} \perp \overline{BC}$

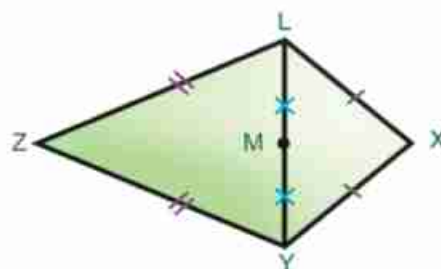
∴ $\overrightarrow{AD}$ bisects both of the base $\overline{BC}$ and ∠ BAC

∴ m (∠ DAC) = m (∠ DAB) = 25°,

D C = $\frac{1}{2}$ BC = $\frac{4}{2}$ = 2 cm.

practice

1 **In the figure opposite**

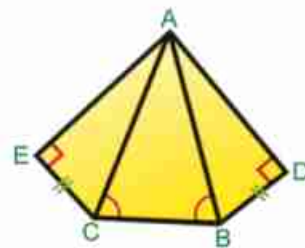
x y = x L , Zy = Z L , L M = Y M

Prove that X, M and Z are on the same straight line

2 **In the figure opposite:**

BD = CE

m (∠ ABC) = m (∠ ACB)

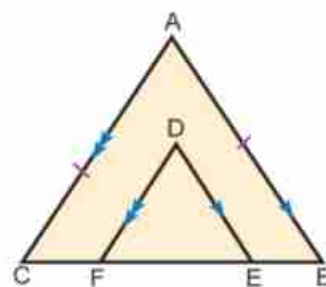
m (∠ D) = m (∠ E) = 90°

Prove that : m (∠ DAB) = m (∠ CAE)

3 **In the figure opposite:**

AB = AC, $\overline{DE}$ \\ $\overline{AB}$

$\overline{DF}$ \\ $\overline{AC}$

Prove that: DE = DF

Second: m (∠ BAC) = m (∠ E D F)

Exercises (4-4)

1 Complete to get a correct statement :

- **A** The vertex angle bisector in the isosceles triangle bisects the base and is ....
- **B** The number of symmetrical axes in the equilateral triangle is ....
- **C** Any point at the axis of a line segment symmetry is at two equal distances from ....
- **D** If the measurement of an angle in the isosceles triangle is $100°$, then the measurement of an angle of the other two = ...°.

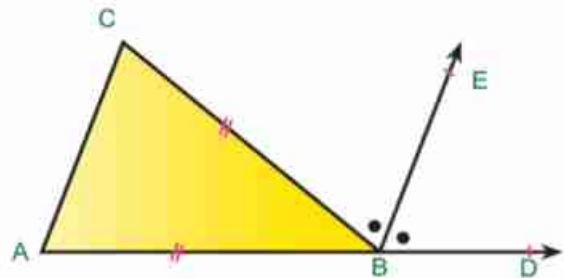
2 Choose the correct answer:

- **A** The number of axes of symmetry in the isosceles triangle = ...
  (0 , 1 , 2 , 3)
- **B** The triangle whose sides lengths are 2cm, $(X + 3)$ and 5cm becomes an isosceles triangle when X = ... cm.
  (1 , 2 , 3 , 4)
- **C** The intersecting point of the medians of a triangle divides each other from the direction of the base in a ratio ....
  (1 : 2 , 2 : 1 , 1 : 3 , 2 : 3)

3 In the figure opposite:

$AB = BC$, $\overrightarrow{BE}$ bisects $\angle CBD$

, $D \in \overrightarrow{AB}$

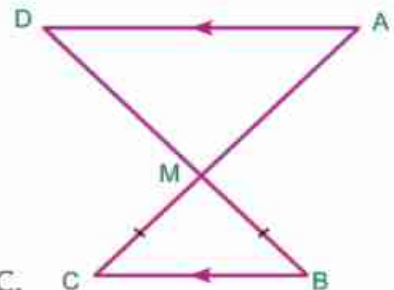
Prove that: $\overrightarrow{BE} \parallel \overline{AC}$

4 In the figure opposite:

$\overline{AC} \cap \overline{BD} = \{M\}$

$\overline{AD} \parallel \overline{BC}$, $MB = MC$

Prove that :

(1) $\triangle$ AMD is an isosceles triangle

(2) The axis of symmetry of $\triangle$ AMD is the same of $\triangle$ BMC.

General Exercises

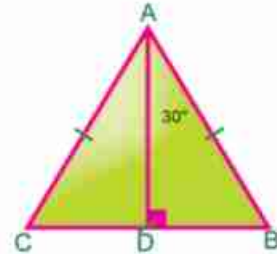
1 **In the figure opposite**

$AB = AC$, $BC = 10cm$,

$m(\angle BAD) = 30°$, $\overline{AD} \perp \overline{BC}$

First: Find the length of $\overline{BD}$, $\overline{AD}$.

Second: How many axes of symmetry are there at ΔABC?

Third: What is the area of ΔABC?

2 **In the figure opposite**

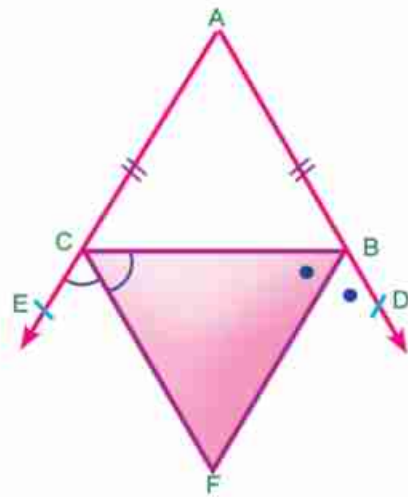
$AB = AC$, $D \in \overrightarrow{AB}$, $E \in \overrightarrow{AC}$

$\overrightarrow{BF}$ bisects $\angle DBC$,

$\overrightarrow{CF}$ bisects $\angle BCE$

Prove that

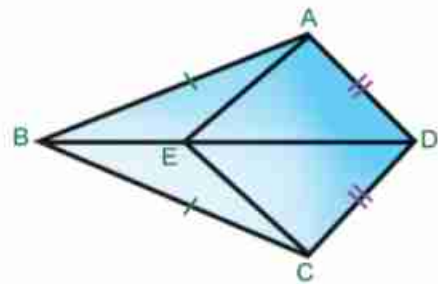
First: ΔBFC is an isosceles triangle

Second: $\overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$

3 **In the figure opposite**

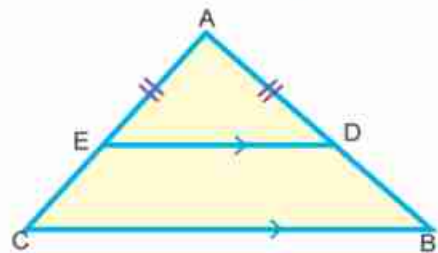
$AB = CB$, $AD = CD$, $E \in \overline{BD}$

Prove that

$\overrightarrow{BD}$ bisects $\angle ADC$

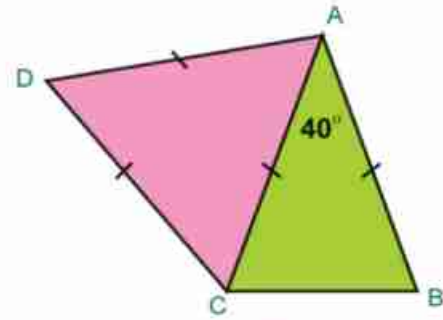
$\overrightarrow{DE}$ bisects $\angle ABC$

4 **In the figure opposite**

$\overline{DE} \parallel \overline{BC}$, $AD = AE$

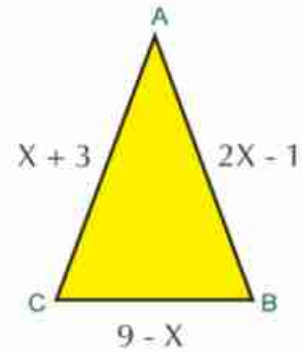
prove that: $AB = AC$.

5. **In the figure opposite**

$AB = AC = AD = CD$

$m(\angle BAC) = 40$

Find: $m(\angle BCD)$

6. **In the figure opposite**

ABC is a triangle which $m(\angle B) = m(\angle C)$

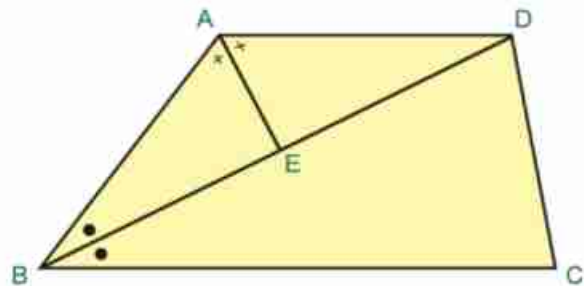
Find: The perimeter of in the triangle

7. **In the figure opposite**

ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$, $\overrightarrow{BD}$ bisects $\angle ABC$, $\overrightarrow{AE}$ bisects $\angle BAD$

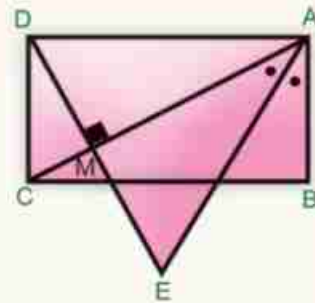
Prove that

A. $AB = AD$

B. $\overline{AE} \perp \overline{BD}$

C. $BE = ED$

Activity

1. Using a ruler and a compass, draw the acute-angle ABC. At the opposite side from $\overrightarrow{BA}$. Draw $\overrightarrow{AE}$ \\ $\overleftrightarrow{BC}$

2. In the opposite figure ABCD is a rectangle in which $\overline{AC}$ is a diagonal, $\overrightarrow{AE}$ bisects $\angle BAC$, $\overline{DE} \perp \overline{AC}$ where $\overrightarrow{AE} \cap \overrightarrow{DE} = \{E\}$ $\overline{AC} \cap \overline{DE} = \{M\}$

 Prove that $DA = DE$.

Unit test

1 **Complete** **the following to get a correct statement :**

A. The base angles in an isosceles triangle are ....

B. The median that is drawn from the vertex of an isosceles triangle is .... and ....

C. In the triangle ABC, AB = AC, m (∠A) = 70° , so m (∠C) =

D. The number of the axes of symmetry in an equilateral triangle = ...

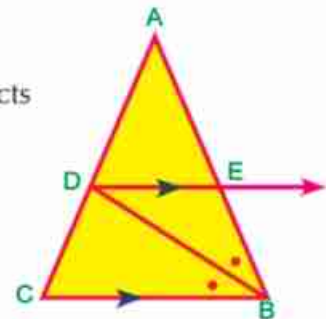
E. The measure of the exterior angle in an equilateral triangle = ...

F. The straight line perpendicular to the midpoint of a line segment is called

2 **In the figure opposite :**

ABC is a triangle in which $\overrightarrow{BD}$ bisects ∠ ABC and intersects $\overline{AC}$ at D, $\overrightarrow{DE} \parallel \overline{CB}$

$\overrightarrow{DE} \cap \overline{AB} = \{E\}$

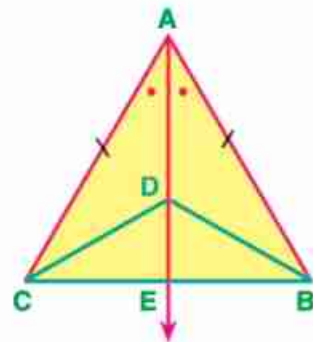
Prove that B E = E D

3 **In the figure opposite** ABC is a triangle in which

AB = AC, $\overrightarrow{AE}$ biscts ∠ BAC,

$\overline{AE} \cap \overline{BC} = \{E\}$

$D \in \overline{AE}$.

Prove that

A. $BE = \frac{1}{2} BC$ B. BD = CD

UNIT FIVE

5

Inequality

Inequality

Think and Discuss

You will learn how

- To define the concept of inequality.
- To define axioms of inequality.

key terms

- Inequality.
- axioms.
- greater than >.
- Less than <.
- equal to

The concept of inequality:

1. Do all the students in your class have the same height?
2. Are there any differences among the measures of acute, right and obtuse angles?

What does this difference mean?

Remark :

An Inequality means that there is a difference in the heights of the students and in the measures of the angles. This difference is represented by the relation of inequality which is used to compare two different numbers.

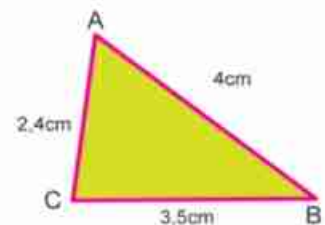
Examples

1. **If:** $\angle ABC$ is an acute angle **then:** m ($\angle ABC$) $< 90^\circ$.
2. **In the figure opposite ,** ABC is a triangle in which:

   AB = 4cm, BC = 3.5cm,

   A C = 2.4cm

   then: AB > BC , BC > AC

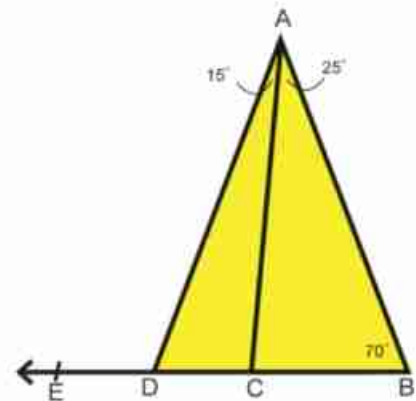
   i.e AB > BC > AC

practice:

In the figure opposite , find: m (∠ ACB), m (∠ ACD) and m (∠ ADE) then complete by using > or < :

m (∠ ADE) .... m (∠ CAD)

m (∠ ADC) .... m (∠ ACB)

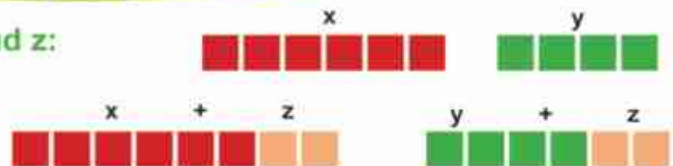
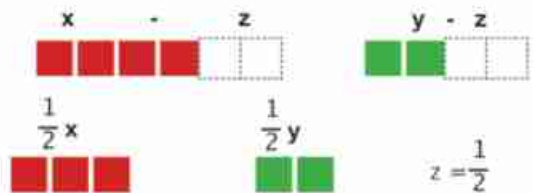
m (∠ ACD) .... m (∠ ABC)

m (∠ ACD) .... m (∠ ADE)

Remark : All the previous relations are called inequalities.

Axioms of inequality

For any given three numbers x, y and z:

x y

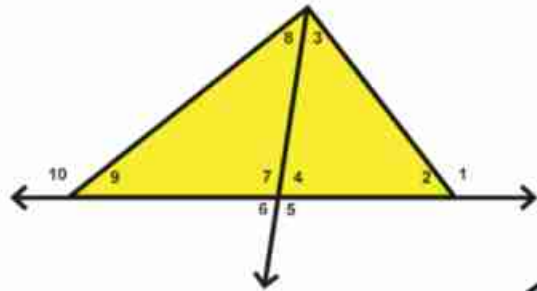
1 **If:** $x > y$
then: $x + z > y + z$

2 **If:** $x > y$
then: $x - z > y - z$

3 **If:** $x > y$, z **is a positive number**
then: $x z > y z$

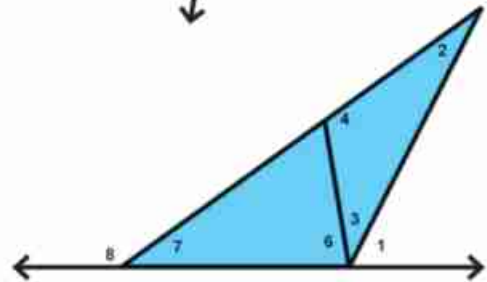
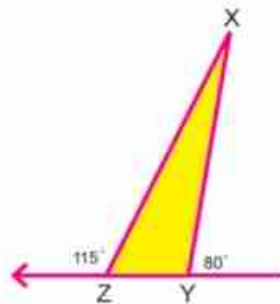
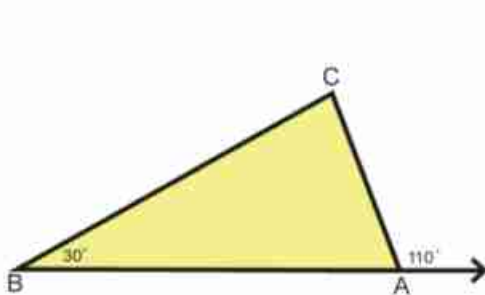
4 **If:** $x > y$, $y > z$
then: $x > z$.

5 **If:** $x > y$, $A > B$
then: $x + A > y + B$

Remember: **The measure of any exterior angle of a triangle is greater than the measure of any interior angle except for the adjacent angle.**

1 **In the figure opposite: which of the following angles has the greatest measure?**

A) ∠1 , ∠3 , ∠4
B) ∠4 , ∠8 , ∠9
C) ∠2 , ∠3 , ∠7
D) ∠7 , ∠8 , ∠10

2 **In the figure opposite , find:**

A) All angles of measures less than m (∠ 1)
B) All angles of measures greater than m (∠ 6)
C) All angles of measures less than m (∠ 4)

3 **Order the measures of the angles in the triangle ABC in an ascending order and the measures of the angles in the triangle X Y Z in a descending order.**

m (∠ ....) < m (∠ ....) < m (∠ ....)   m (∠ ....) > m (∠ ....) > m (∠ ....)

4 **In the figure opposite:** $C \in \overleftrightarrow{AB}$, $D \in \overleftrightarrow{AB}$

If: A B > C D

then: A C .... B D

Example

In the figure opposite :

$m(\angle ACB) > m(\angle ABC)$, $DB = DC$

Prove that : $m(\angle ACD) > m(\angle ABD)$

Given: $m(\angle ACB) > m(\angle ABC)$, $DB = DC$

Required to prove: $m(\angle ACD) > m(\angle ABD)$

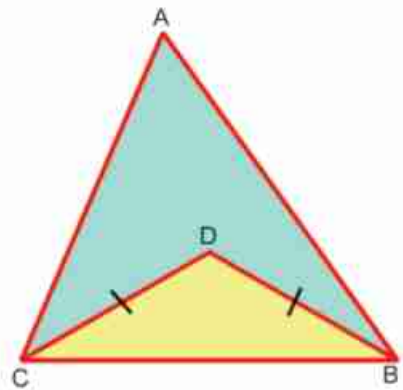
R.T.P: $\because DB = DC$

$\therefore m(\angle DCB) = m(\angle DBC)$ **(1)**

$\because m(\angle ACB) > m(\angle ABC)$ **(2)**

$\therefore$ By subtracting **(1)** from **(2)** , we get:

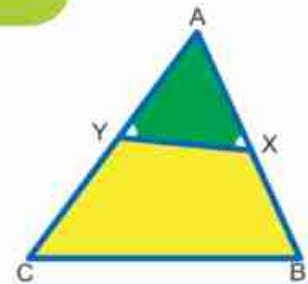
$m(\angle ACB) - m(\angle DCB) > m(\angle ABC) - m(\angle DBC)$

$\therefore m(\angle ACD) > m(\angle ABD)$ **Q.E.D**

Exercises (5-1)

1 **In the figure opposite :**

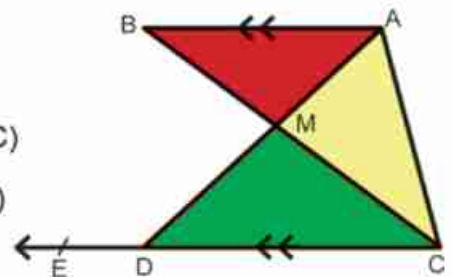
ABC is a triangle in which $AC > AB$, $x \in \overline{AB}$

$y \in \overline{AC}$,where $m(\angle AXY) = m(\angle AYX)$

Prove that: $YC > XB$

2 **In the figure opposite :** $\overline{AB} // \overrightarrow{CD}$,

$\overline{AD} \cap \overline{CB} = \{M\}$, $E \in \overrightarrow{CD}$, $E \notin \overline{CD}$

Prove that: **A** $m(\angle ACD) > m(\angle ABC)$

B $m(\angle ADE) > m(\angle ABC)$

3 M is a point in a triangle ABC ,

Prove that: $m(\angle AMB) > m(\angle ACB)$

Comparing the measures of the angles of a triangle

Think and Discuss

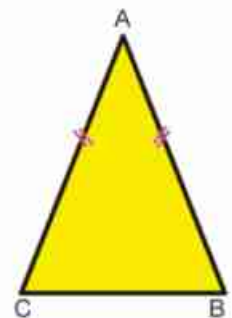
You will learn how

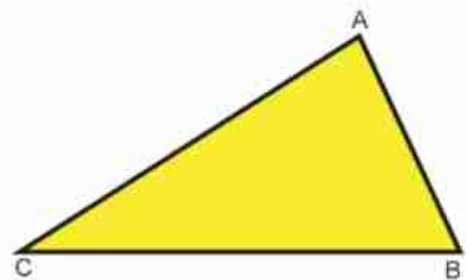
- To compare the measures of angles in a triangle.

key terms

- Angle.
- Measure of an angle.
- The greatest angle in a triangle.
- The smallest angle in a triangle.
- The largest side of a triangle.
- The smallest side of a triangle...

Activity

1 **In the figure opposite:** ABC is an isosceles triangle in which AB = AC

- Fold the triangle to make the vertex B congruent to vertex C. What do you observe regarding to the measures of the angles B, C which are opposite to the two equal sides $\overline{AC}$, $\overline{AB}$?
- Fold the triangle to make the vertices A, C congruent. what do you observe regarding to the measures of the two angles opposite to the two unequal sides $\overline{BC}$, $\overline{AB}$?
- Does the difference in the lengths of the two sides in a triangle lead to a difference in the measures of their two opposite angles?

2 Draw the scalene triangle. ABC Flip the triangle to make the vertex A coincide the vertex B. What do you observe regarding to the measures of the two angles A, and B that are opposite to the two unequal sides, $\overline{BC}$, $\overline{AC}$.

- Repeat the previous steps to make the vertex B coincide vertex C. what do you observe?

Are there any equal angles in measures in that triangle?

Notice that : In a triangle, if the sides are unequal in length, the measures of the opposite angles are unequal.

Activity

Draw the scalene triangle ABC, then measure the lengths of its 3 sides and the measures of the opposite angles, then complete the following table::

Lengths of sides	Measures of the opposite angles
AB = ... cm	$m(\hat{C})$ = ... °
BC = ... cm	$m(\hat{A})$ = ... °
CA = ... cm	$m(\hat{B})$ = ... °

What do you observe?

Theorem (3)

(Angle - Comparison Theorem)

In a triangle, if two sides have unequal lengths, then the longer is opposite to the angle of the greater measure.

Given: ABC in which $AB > AC$

R.T.P: $m(\hat{ACB}) > m(\hat{ABC})$

Construction: take $D \in \overline{AB}$ where $AD = AC$

proof: in $\triangle ACD$, $AD = AC$

$\because m(\hat{ACD}) = m(\hat{ADC})$ **(1)**

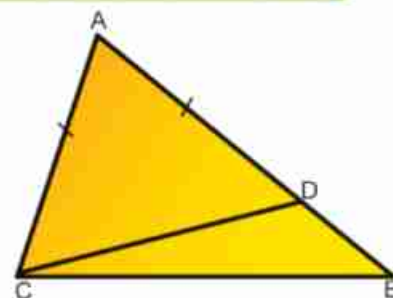
$\hat{ADC}$ is an exterior angle of $\triangle BDC$

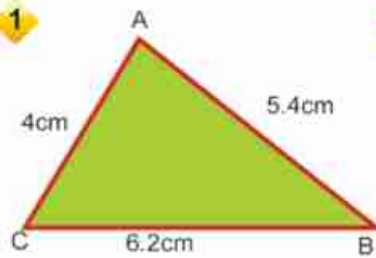
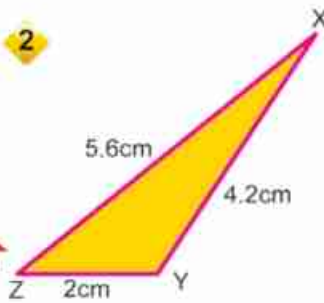
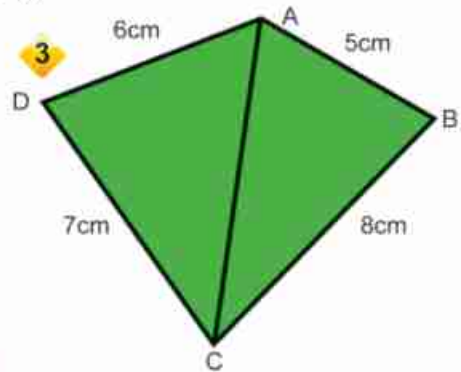
$m(\hat{ADC}) > m(\hat{B})$ **(2)**

from **(1)**, **(2)**

$m(\hat{ACD}) > m(\hat{B})$

$m(\hat{ACB}) > m(\hat{ACD})$

$m(\hat{ACB}) > m(\hat{ABC})$ **Q.E.D**

Practice

In each of the following figures, complete using (>, <))

1		2		3	
m ($\angle$ A)	m ($\angle$ B)	m ($\angle$ z)	m ($\angle$ y)	m ($\angle$ BAC)	m ($\angle$ BCA)
m ($\angle$ A)	m ($\angle$ C)	m ($\angle$ x)	m ($\angle$ y)	m ($\angle$ DAC)	m($\angle$ DCA)
m ($\angle$ B)	m ($\angle$ C)	m ($\angle$ z)	m ($\angle$ x)	m ($\angle$ BAD)	m ($\angle$ BCD)

Remark : The measure of the greatest angle in the triangle > 60°

The measure of the smallest angle in the triangle is < 60° why?

Example

In the figure opposite :

ABC is a triangle in which AB > BC > CA

Prove that: m ($\angle$ C) > m ($\angle$ A) > m ($\angle$ B)

Given: AB > BC > CA

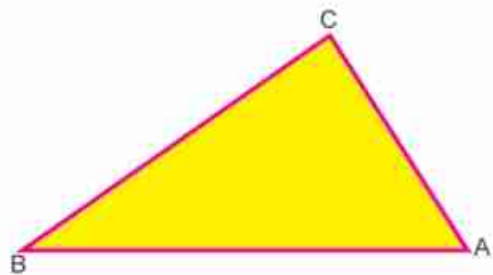
R.T.P: m ($\angle$ C) >m ($\angle$ A) > m ($\angle$ B)

Proof: In Δ A B C

$\because$ A B > B C $\quad$ $\therefore$ m ($\angle$ C) > m ($\angle$ A) $\quad$ **(1)**

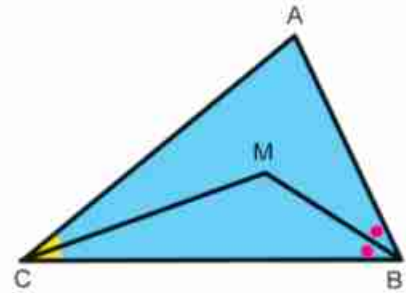
$\because$ B C > C A $\quad$ $\therefore$ m ($\angle$ A) > m ($\angle$ B) $\quad$ **(2)**

from **(1)**, **(2)** and using the axioms of inequality :

m ($\angle$ C) > m ($\angle$ A) > m ($\angle$ B)

Remember : In a triangle, the longest side in length is opposite to the greatest angle in measure while the shortest side in length is opposite to the smallest angle in measure.

Example

In the figure opposite :

ABC is a triangle where $\overrightarrow{BM}$ bisects ∠ A B C, and $\overrightarrow{CM}$ bisects ∠ ACB If: M C > MB

Prove that: m (∠ ABC) > **m** (∠ ACB)

Given : $\overrightarrow{BM}$ bisects ∠ A B C, $\overrightarrow{CM}$ bisects ∠ ACB , MC > MB.

R.T.P: Prove that (∠ ABC) > **m** (∠ ACB)

Proof: in Δ MBC

∵ MC > MB ∴ **m** (∠ MBC) > **m** (∠ MCB) **(1)**

In Δ ABC

∵ $\overrightarrow{BM}$ bisects ∠ ABC ∴ **m** (∠ MBC) = $\frac{1}{2}$ **m** (∠ ABC) **(2)**

∵ $\overrightarrow{CM}$ bisects ∠ ACB ∴ **m** (∠ MCB) = $\frac{1}{2}$ **m** (∠ ACB) **(3)**

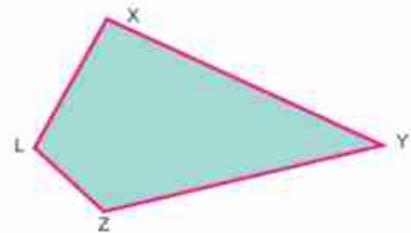
∴ from **(1), (2), (3) :** $\frac{1}{2}$ **m** (∠ ABC) > $\frac{1}{2}$ **m** (∠ ACB) Using the axioms of inequality

∴ m (∠ ABC) > m (∠ ACB) Q.E.D

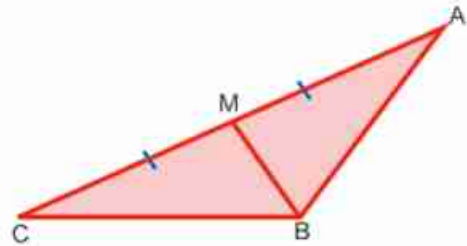
Exercise (5-2)

1. Δ ABC in which AB = 2.7cm, BC = 8.5cm, AC = 6cm. Order the measures of the angles of the triangle ascendingly.

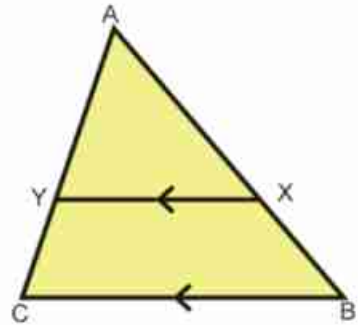
2. In the figure opposite X Y > X L, Y Z > Z L

   Prove that: m (∠ XLZ) > m (∠ XYZ)

3 **In the figure opposite :**

$\overline{BM}$ is a median in the triangle ABC , BM < AM

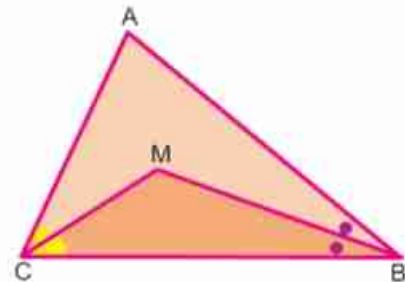
Prove that : ∠ ABC is an obtuse angle.

4 **In the figure opposite :**

ABC is a triangle , AB > AC , $\overline{XY}$ // $\overline{BC}$

Prove that: m (∠ AYX) > (∠ AXY)

5 **In the figure opposite:**

A B C is a triangle، $\overrightarrow{BM}$ bisects ∠ A B C ,

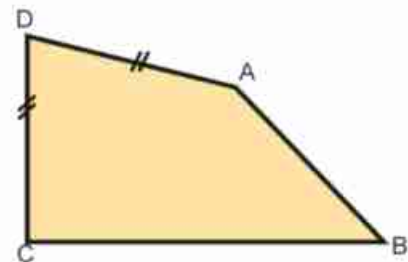
$\overrightarrow{CM}$ bisects ∠ ACB.if AB > AC

Prove that : m (∠ MCB) > m (∠ MBC)

6 **In the figure opposite :**

ABCD is a quadrilateral in which AD = DC,

BC > AB

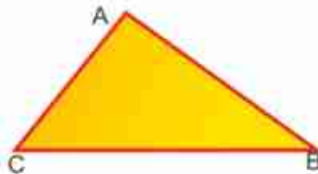
Prove that :

m (∠ A) > m (∠ C)

7 **ABCD is a quadrilateral in which $\overline{AB}$ is the longest side in length. $\overline{CD}$ is the shortest one. Prove that m (∠ BCD) > m (∠ BAD)**

Comparing the lengths of sides of a triangle

Think and Discuss

Activity 1 **The figure opposite:** ABC is a triangle of unequal measures of angles.

- Fold the triangle to make the vertex A coincide vertex B what do you observe regarding to the lengths of the two sides $\overline{BC}$ and $\overline{AC}$, which are opposite to the two unequal angles A and B?
- Repeat the same previous steps to make vertex B congruent to vertex C. What do you observe?
- When vertex C is coincide to vertex A, what do you observe?
- Are there any equal sides in lengths in that triangle?

Remark : **If the measures of the angles in a triangle are unequal, then the lengths of its sides which are opposite to the angles are unequal.**

Activity 2 Draw the triangle ABC where its angles are unequal in measure then measure the lengths of opposite sides to the angles and complete the following table:

the measures of the angles	the lengths of the opposite sides
m (∠ A) = ...°	B C = ... cm
m (∠ B) = ...°	C A = ... cm
m (∠ C) = ...°	A B = ... cm

What do you observe?

- Is the greatest angle in measure opposite to the longest side in length? Is the smallest angle in measure opposite to the shortest side in length?
- Is it possible to order the lengths of the sides in the triangle in an ascending or descending order in terms of the measures of the opposite angles?

You will learn how

- To compare the measures of sides in a triangle..

key terms

- The longest side of a triangle.
- The shortest side of a triangle.
- the greatest angle of a triangle.
- the smallest angle of a triangle.
- The perpendicular line segment.

Theorem (4)

(Side - Comparison Theorem)

In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Given : In ΔABC $m(\angle C) > m(\angle B)$

R. T. P : $AB > AC$

Proof : $\therefore$ $\overline{AB}$, $\overline{AC}$ are line segments

$\therefore$ one of the following cases should be verified:

(1) $AB < AC$ (2) $AB = AC$ (3) $AB > AC$

If not $AB > AC$

Either $AB = AC$ **or** $AB < AC$

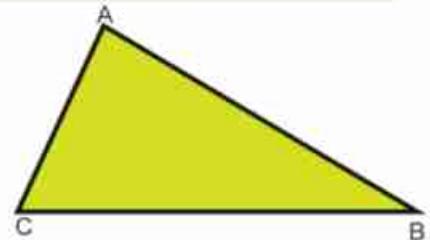
if $AB = AC$, then **m** $(\angle C) =$ **m** $(\angle B)$

Again this contradicts the given where $m(\angle C) > m(\angle B)$

and if $AB < AC$, then $m(\angle C) < m(\angle B)$. According to the theorem above.

Again this contradicts the given, where

$m(\angle C) > m(\angle B)$

$\therefore AB > AC$ Q.E.D

practice

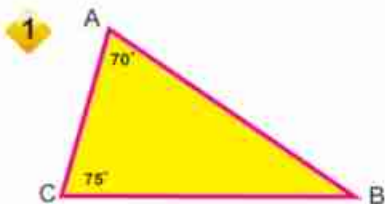
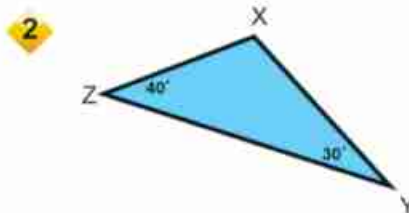
In the following figures , complete using > , < or =:

1

AB ...... AC

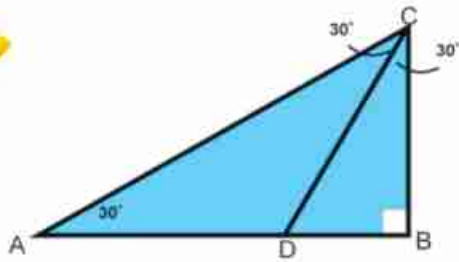
AB ...... BC

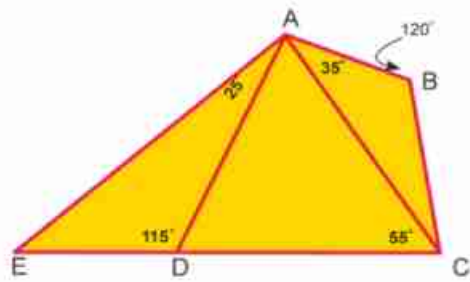
AC ...... BC

2

XY ...... XZ

YZ ...... XY

YZ ...... XZ

3

A C .... B C
B C .... D B
A C .... B D
C D .... A C

4

B C .... A B
C D .... C A
A D .... A E
C D .... A D

Corollaries :

Corollary (1)

In the right - angled trinagle, the hypotenuse is the longest side.

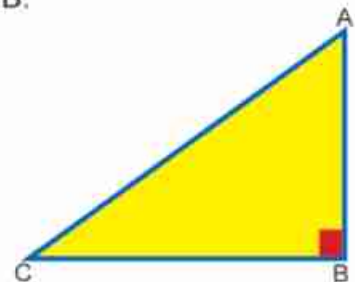
In the figure opposite Δ ABC is a right - angled triangle at B.

$\because \angle A$ acute $\quad \therefore m(\angle B) > m(\angle A)$

AC > BC

$\because \angle C$ acute $\quad \therefore m(\angle B) > m(\angle C)$

AC > A B

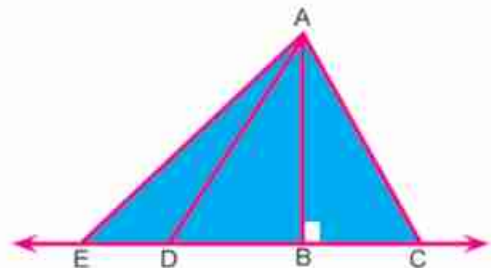
$\therefore \overline{AC}$ is the longest side Q.E.D.

Remark : In the obtuse angled triangle, the side opposite to the obtuse angle is the longest side in the triangle .

Let's think

A C > A B. Why?

A D > A B. Why?

A E > A B. Why?

Is the length of the right leg in the right angled-triangle is shorter than the length of the hypotenuse? Why?

Corollory (2)

The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given striaght line.

Definition : The distance between any point and a given straight line is the length of the perpendicular line segment drawn from the point to the given line.

Example

in the figure opposite: ABC is a triangle , $E \in \overrightarrow{BA}$

$\overrightarrow{AD}$ \\ $\overline{BC}$, **m** $(\angle CAD) = 35^\circ$

m $(\angle DAE) = 75^\circ$

Prove that : $AC > AB$

Given that: $\overrightarrow{AD} \parallel \overline{BC}$, **m** $(\angle EAD) = 75^\circ$, **m** $(\angle DAC) = 35^\circ$

R.T.P: $AC > AB$

Proof: $\because \overrightarrow{AD} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ **m** $(\angle B) = m(\angle EAD) = 75^\circ$ — **Corresponding angles (1)**

$\because \overrightarrow{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

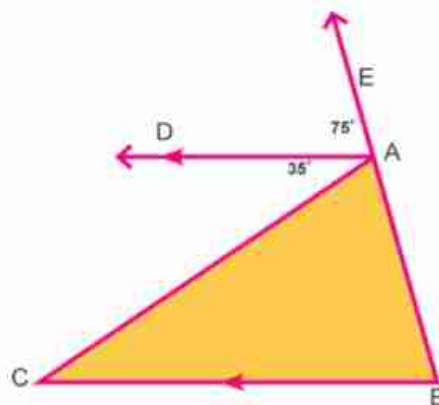
$\therefore m(\angle ACB) = m(\angle DAC) = 35^\circ$ — **alternate angles (2)**

From **(1) and (2)** :

in ΔABC

$m(\angle ABC) = 75^\circ$, $m(\angle ACB) = 35^\circ$

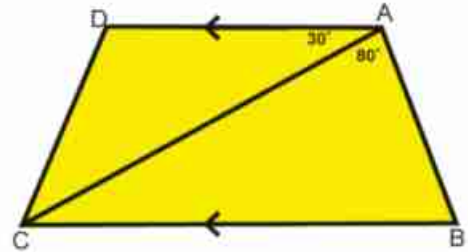
i.e. $m(\angle ABC) > m(\angle ACB)$

$\therefore AC > AB$ — **Q.E.D**

Exercise (5-3)

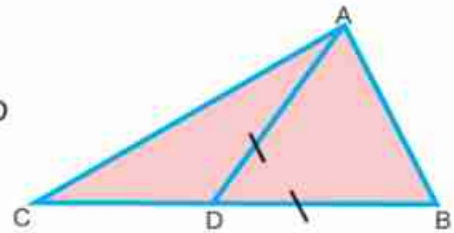
1 ΔABC in which $m(\angle A) = 40^\circ$, $m(\angle B) = 75^\circ$, Order the lengths of sides of the triangle descendingly.

2 **In the figure opposite :**

$\overrightarrow{AD} \parallel \overline{BC}$, $m(\angle BAC) = 80^\circ$

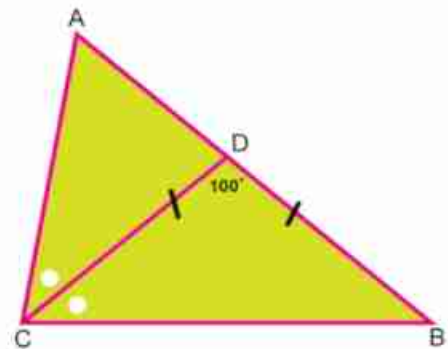
$m(\angle DAC) = 30^\circ$. Prove that: $BC > AB$

3 **In the figure opposite :**

in ΔABC is a triangle, $D \in \overline{BC}$ where $BD = AD$

prove that : $BC > AC$

4 **In the figure opposite :**

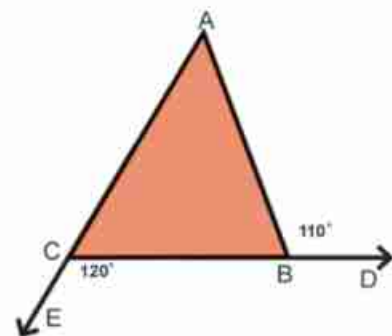
ABC is a triangle, $\overrightarrow{CD}$ bisects $\angle C$ and intersects $\overline{AB}$ at point D.

$m(\angle BDC) = 100^\circ$, $DB = DC$

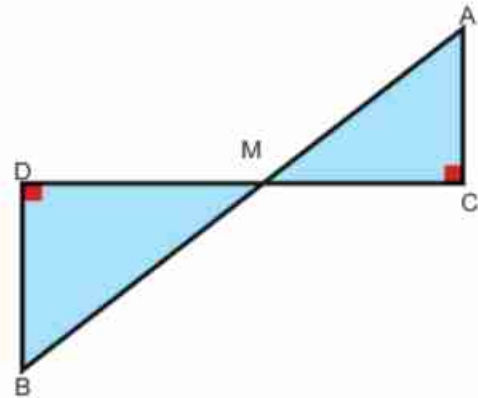
Prove that : $AC > DB$.

5 **In the figure opposite :**

ABC is a triangle, $D \in \overrightarrow{CB}$, $E \in \overrightarrow{AC}$

$m(\angle ABD) = 110^\circ$, $m(\angle BCE) = 120^\circ$

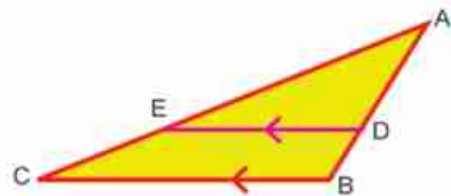
Prove that: $AB > BC$.

6 **In the figure opposite:**

$\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$, $\overline{BD} \perp \overline{CD}$

prove that : $AB > CD$

7 **In the figure opposite :**

ABC is an obtuse-angled triangle at B

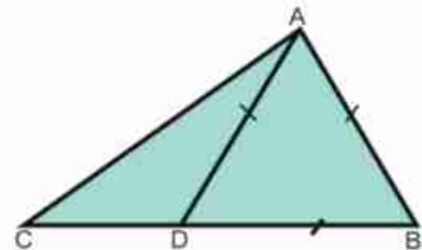
$\overline{DE} \mathbin{//} \overline{BC}$

Prove that: $AE > AD$

8 ABC is a triangle, $\overrightarrow{CD}$ bisects $\angle C$, $\overrightarrow{CD} \cap \overline{AB} = \{D\}$, prove that $BC > BD$

9 ΔABC in which **m** $(\angle A) = (5x + 2)^\circ$, **m** $(\angle B) = (6x - 10)^\circ$, **m** $(\angle C) = (x + 20)^\circ$,
order the lengths of the sides of the triangle ascendingly.

10 **In the figure opposite :**

ABC is a triangle, $D \in \overline{BC}$, $AB = AD = BD$

Prove that : $BC > AC$

11 ABC is a right-angled triangle at B , $D \in \overline{AC}$, $E \in \overline{BC}$, where $AD = BE$
Prove that: **m** $(\angle CED) >$ m $(\angle CDE)$

Triangle inequality

Think and Discuss

Activity

By using your ruler and compass, try to draw the triangle ABC where :

1. $AB = 4$ cm , $BC = 5$ cm , $AC = 6$ cm
2. $AB = 6$ cm , $BC = 3$ cm , $AC = 2$ cm
3. $AB = 9$ cm , $BC = 4$ cm , $AC = 3$ cm
4. $AB = 8$ cm , $BC = 3$ cm , $AC = 5$ cm

In which of the previous cases were you able to draw the triangle? What do you conclude?

You will learn how

- To define the triangle inequality .

key terms

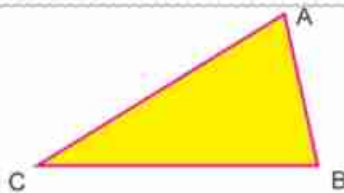
- inequality .
- triangle inequality.

Fact : For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

i.e. : In any triangle ABC :

$AB + BC > AC$

$BC + CA > AB$

$AB + AC > BC$

for example: the numbers 5, 3 and 9 are not valid to be the lengths of a triangle because the sum of the smallest two numbers = 3 +5 = 8 , 8< 9. Therefore, the inequality of the triangle is not verified.

Examples

ABC is a triangle , If $AB = 10$ cm, $BC = 8.5$ cm

Find the interval which the length of side $\overline{AC}$ belongs to.

Solution

$AC < AB + BC$ $\qquad AC < 18.5$ **(1)**

However, $AC + BC > AB$ **triangle inequality**

$AC > AB - BC$ $\qquad AC > 1.5$ **(2)**

From (1) , (2) $\quad 18.5 > AC > 1.5$

$AC \in]1.5 \ , \ 18.5[$

Practice

Find the interval which the third side belongs to in each of the follwing triangles. If the lengths of the other two sides were as follows:

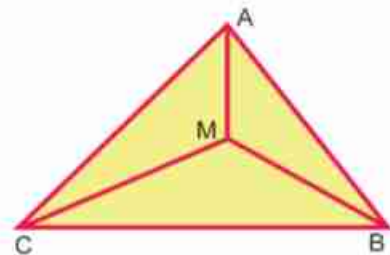
A 6 cm, 9 cm B 5 cm, 12 cm C 7 cm, 15 cm D 2.9 cm, 3.2 cm

Exercise (4-4)

1. If the lengths of two sides in an isosceles triangle were 5cm, 12cm , Find the length of the third side (State the reason)

2. Which of the following groups are valid in drawing a triangle?
   - A 5 cm, 7 cm, 8 cm
   - B 4 cm, 9 cm, 3 cm
   - C 10 cm, 6 cm, 4 cm
   - D 15 cm, 17 cm, 30 cm.

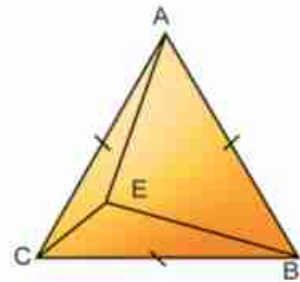
3. Prove that the length of any side in a triangle is less than half of the perimeter of the same triangle.

4. **In the figure opposite:**
   ABC is a triangle in which M is a point inside it. Prove that:
   $MA + MB + MC > \frac{1}{2}$ perimeter of the triangle ABC

5. **prove that :** the sum of the lengths of the two diagonals in a convex quadrilateral is less than its preimeter .

General Exercises

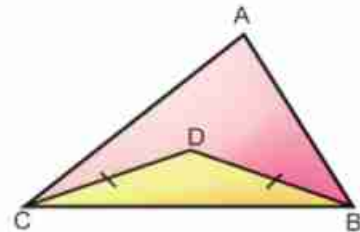
1 In the figure opposite : ABC is an equilateral triangle where E is a point inside it **m** (ECB) > **m** (EBC).

First: prove that : m ($\hat{ABE}$) > **m** ($\hat{ACE}$).

Second: m (A) > **m** ($\hat{ABE}$) > **m** ($\hat{ACE}$).

2 In the figure oppsoite :

D B = D C .

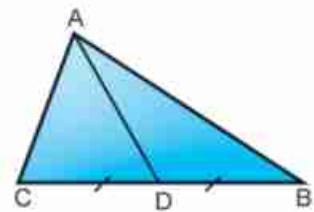
m ($\hat{ABC}$) > **m** ($\hat{ACB}$)

Prove that: m ($\hat{ABD}$) > **m** ($\hat{ACD}$)

3 A B C is a triangle in which AB = 6cm, A C = 7cm, B C = 8cm.

Order the measures of its angles ascendingly.

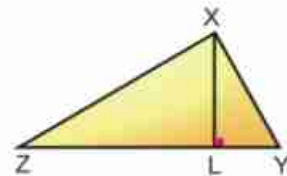
4 In the figure opposite :

AB > AC , DB = DC

Prove that: **m** ($\hat{BAD}$) < **m** ($\hat{CAD}$) .

5 In the figure opposite :

X Z > X Y

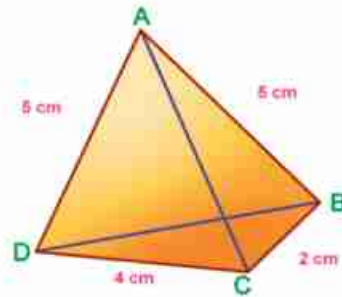
$\overline{XL} \perp \overline{ZY}$

prove that **m** ($\hat{LXZ}$) > **m** ($\hat{LXY}$)

6 **In the figure opposite :**

ABCD is a quadrilateral in which

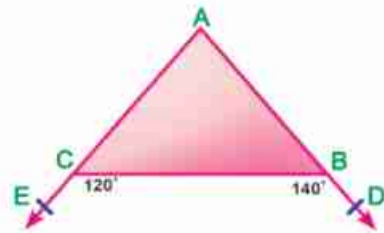
A B = AD = 5cm,

BC = 2 cm, DC = 4cm.

Prove that: **m** ($\hat{ABC}$) > **m** ($\hat{ADC}$)

7 **In the figure opposite :**

m ($\hat{DBC}$) = 140°

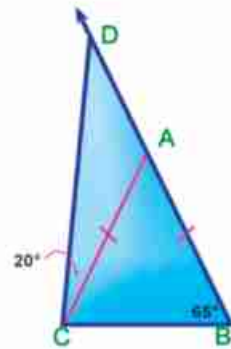
m ($\hat{ECB}$) = 120°

Prove that: C B > A B

8 **In the figure opposite :**

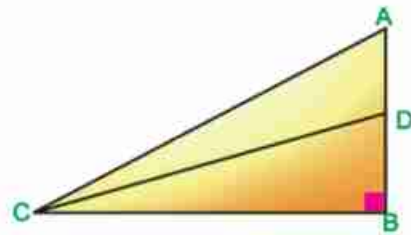
AB = AC

m ($\hat{ABC}$) = 65°

m ($\hat{ACD}$) = 20°

Prove that: AB > AD

9 **In the figure opposite :**

m ($\hat{B}$) = 90°

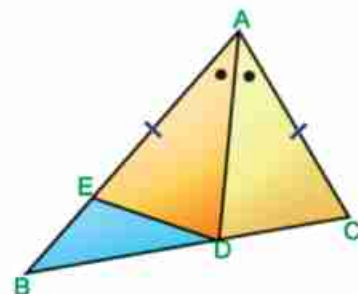
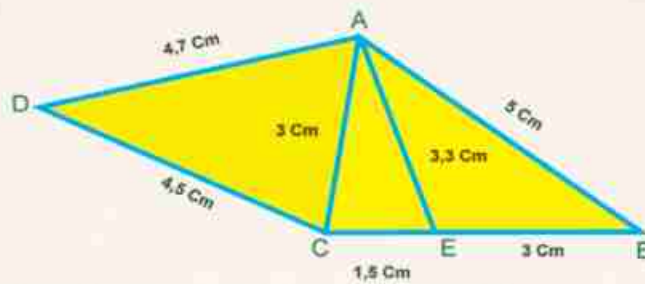
Prove that: A C > D C

10 **In the figure opposite :**

A C > D C , **m** ($\hat{CAD}$) = **m** ($\hat{BAD}$)

A E = A C

Prove that: **A** DE = DC

B **m** ($\hat{BAD}$) > **m** ($\hat{ADC}$)

C BD > DC .

Activity

1 **In the given figure complete using** (< or >)

A m (∠D A C) .... m (∠A C D)

B m (∠A E C) .... m (∠E C A)

C m (∠A B E) .... m (∠E A B)

D m (∠C D A) .... m (∠D A C)

E m (∠A E B) .... m (∠E A C)

2 In the traingle ABC , AB = 6cm , BC = 9cm

then AC ∈] .... , ...[

3 **In the** triangle ABC : **m** (∠ A) = (9x)°, **m** (∠ B) = (6x - 17)°

m (∠ C) = (7 x - 1)°

Order the lengths of the sides of the triangle ascendingly.

Unit test

1 **Complete the fllowing to make each statement true:**

A. In a triangle , the smallest angle in measure is opposite to ....

B. In Δ ABC: if m ($\angle$ A) = 70°, m ($\angle$B) = 30°, then the longest side in length is ....

C. If the lengths of two sides in an isosceles triangle were : 3 cm, 7cm then , the length of the third side =....... cm

D. Δ ABC in which: m ($\angle$ A) = 100°, then the greatest side length is ....

E. Δ ABC in which AB = 3cm, BC = 5cm, then AC $\in$]......, ...[

F. The longest side length in the right-angled triangle is ....

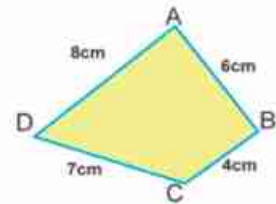
2 **In the figure opposite :**

ABCD is a quadrilateral in which

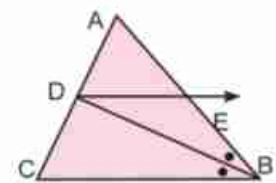
AB = 6cm , BC = 4cm,

CD = 7cm , DA = 8cm

Prove that:

m ($\angle$ BCD) > m ($\angle$ BAD)

3 **In the figure opposite :**

A B C is a triangle, $\overrightarrow{BD}$ bisects $\angle$ B, $\overline{BD} \cap \overline{AC}$ = {D}, $\overrightarrow{DE}$ // $\overline{CB}$ and intersects $\overline{AB}$ at E

Prove that: A B > A D

4 **In the figure opposite :**

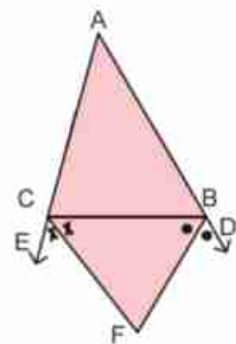
Δ ABC in which AB > AC, D $\in \overrightarrow{AB}$, E $\in \overrightarrow{AC}$

$\overrightarrow{BF}$ bisects $\angle$ DBC , $\overrightarrow{CF}$ bisects $\angle$ BCE

$\overrightarrow{BF} \cap \overrightarrow{CF}$ = {F}

Prove that :

A. m ($\angle$FBC) > m ($\angle$BCF)

B. CF > BF

Model tests on Algebra and statistics (2nd preparatory)

Model (1)

First question : *complete the following :*

(1) The S.S of the equation $(\chi^2+3)(\chi^3+1)=0$ is .... : $\chi \in \mathbb{R}$

(2) If the lowest boundary of a set is 10 and the upper boundary is χ and its centre is 15, then $\chi =$ ....

(3) $]-2, 2] \cup \{-2, 0\} =$ ....

(4) A cube whose volume is 8 cm^3, then the sum of the lengths of all its edges equals .... cm.

(5) The multiplicative inverse of the number $\sqrt{3}+\sqrt{2}$ is ....(in the simplest form).

Second question : *Choose the correct answer from those given.*

(1) If the radius of a sphere is 6 cm , then its volume is ....

a. 6π cm^3 b. 36π cm^3 c. 72π cm^3 d. 288π cm^3

(2) If the point (a,1) satisfies the relation x+y=5, then a= ....

a. 1 b. -4 c. 4 d. 5

(3) $(2\sqrt[3]{2})^3 =$ ....

a. 4 b. 8 c. 16 d. 40

(4) The median of the values 34 , 23 , 25 , 40 , 22 , 4 is ....

a. 22 b. 23 c. 24 d. 25

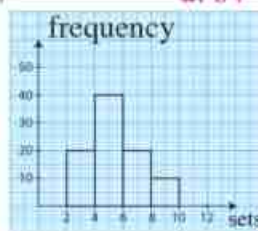
(5) If the arithmetic mean of the values 27 , 8 , 16 , 24 , 6 , k is 14 then k = ...

a. 3 b. 6 c. 27 d. 84

(6) In the opposite figure

the value of the mode = ...

a. 4 b. 5

c. 6 d. 40

question (3)

(a) Find the value of $\sqrt{18}+\sqrt[3]{54}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{16}$

(b) If $\chi=\frac{3}{\sqrt{5}-\sqrt{2}}$, $y=\sqrt{5}-\sqrt{2}$, prove that: χ , y are two conjugate

question (4)

(a) The area of a square is 1089 cm^2. Find the length of its diagonal.

(b) Find the S.S of the inequality $\frac{3\chi+1}{6}<\chi+1<\frac{\chi+4}{2}$ in $\mathbb{R}$, then represent it on the number line.

question (5)

(a) The radius length of the base of a right circular cylinder , its base radius length is 4 $4\sqrt{2}$ cm and its height is 9 cm find its volume in terms of π and if its volume equals the volume of a sphere find the radius length of the sphere.

(b) Find the arithmetic mean of the following frequency distribution.

The set	5 –	15 –	25 –	35 –	45 –	total
Frequence	7	10	12	13	8	50

Model (2)

Qvestion one : Complete the following :

(1) The additive inverse of the number $-\sqrt{3}-\sqrt{5}$ is

(2) $(\sqrt{8}+\sqrt{2})(\sqrt{8}-\sqrt{2})=$ ....

(3) The conjugate of the number $\frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$ is ....

(4) If the volume a sphere is $\frac{9}{2}\pi$ cm³ then its diameter length is .... cm.

(5) $\{3,4\}-\{3,5\}=$ ....

Question (2) : Choose the correct answer from those given :

(1) If the volume of a cube is 27 cm^3 then the area of one of its faces is ....

a. 3 cm^2 b. 9 cm^2 c. 36 cm^2 d. 54 cm^2

(2) If the mode of the set of values 4 , 11 , 8 , 2 χ is 4 then χ = ...

a. 2 b. 4 c. 6 d. 8

(3) If the arithmetic mean of the set of values 18 , 23 , 29 ,2k-1 , k is 18 then k = ...

a. 1 b. 7 c. 29 d. 90

(4) If the lowest limit of a set is 4 and the upper limit is 8 then its centre is ....

a. 2 b. 4 c. 6 d. 8

(5) A right circular cylinder, the radius of its base is r, its height equals the length of the dimeter, then its volume= ....

a. πr^3 b. πr^2 c. $2\pi r^3$ d. $2r^3$

(6) The sloution set in R of the equation $X(X^2-1)=0$ is ....

a. {0} b. {1} c. {-1} d. {0,-1,1}

question (3)

(a) Reduce to the simplest form. $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

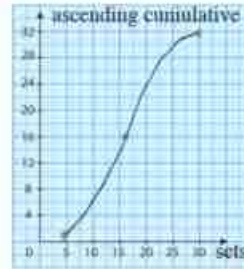
(b) Prove that $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=0$

question (4)

(a) Find the S.S of the inequality $-2 < 3\chi + 7 \leq 10$ in $\mathbb{R}$, then represent the in terval of solution on the number line.

(b) If $\chi = \sqrt{2}+\sqrt{3}$ find the value of $\chi^4 - 2\chi^2 + 1$

question (5)

(a) The opposite figure represents the ascending cumulative curve of the marks 32 students in one test complete the median mark = ...

(b) Find the arithmetic mean of the following frequency distribution.

The set	5 –	15 –	25 –	35 –	45 –	total
Frequence	4	5	6	3	2	20

Model Test (3)

"Merge students"

Questions (1) **Complete each of the following:**

1) The conjugate of the number $\sqrt{3}+\sqrt{2}$ is ....

2) $\sqrt{18}+\sqrt{54}-3\sqrt{2}$ = ...

3) The mode for the number 3 , 5 , 3 , 4 , 3 is ....

4) The median of the values 2 , 3 , 5 , 7 , 9 is ....

5) The solution set of the equation $x^2 + 9 = 0$ in R is ....

Questions (2) **Choose correct answer:**

1) The arithmetic mean for the values 9 , 6 , 5 , 14 , 1 is ....

(a) 7 (b) 3 (c) 5 (d) 9

2) The simplest form of the expression $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ is ....

(a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$

3) The additive inverse of the number - $\sqrt{5}$ is ....

(a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) -5

4) $[3, 5] - \{3, 5\} =$ ....

(a) $]3, 5[$ (b) $[3, 5[$ (c) φ (d) $]3, 5]$

5) A cube of volume 64 cm^3, then the edge length is .... cm.

(a) 4 (b) 8 (c) 16 (d) 64

Questions (3) **Match from the column (A) to the suitable one from the column (B):**

(A)	(B)
1) The solution set of the equation $x^2 - 25 = 0$ R is	$[0, 2]$
2) $[-3, 2] \cap [0, 2] =$	7
3) If the order of the median is the fourth, than the number of the values is	$\{5, -5\}$
4) $\sqrt{3}$ is an................. number.	(number line: closed segment from 3 to 7)
5) The solution set of the inequality $3 \leq x \leq 7$ on the number line is	Irrational

Questions (4) **Put (√) for the correct statement, (x) for the incorrect one:**

1) The arithmetic mean for the a set of values = the sum of these values ÷ its number. ( )

2) If $x = \sqrt{13} - \sqrt{7}$, $y = \sqrt{13} + \sqrt{7}$, then x , y are two conjugate numbers. ( )

3) The irrational number $\sqrt{7}$ lies between 2 and 3 ( )

4) $\sqrt{75} - 2\sqrt{27} = 7\sqrt{3}$ ( )

5) The simplest from of the number $\frac{1}{\sqrt{5}}$ is $\frac{\sqrt{5}}{5}$ ( )

Questions (5) **First: Complete:**

If the lower limit of a set is 4, the upper limit is 8, then its centre = $\frac{\ldots\ldots + \ldots\ldots}{2}$ = ...

Second : Complete

The following table to obtain the arithmetic mean of the following frequency distribution:

Sets	5-	15-	25-	35-	45-	Total
Frequency	7	10	12	13	8	50

Sets	The centre of the set x	Frequency (F)	F x X
5 –	10	7	10 x 7 = 70
15 –	20	10	20 x 10 = ...
25 –	....	....	.... x 12 = ...
35 –	....	....	.... x 13 = ...
45 -	....	....	.... x 8 = ...
The sum	....	50	....

The arithmetic mean = $\frac{\Sigma(F \times X)}{\Sigma(F)} = \frac{..........}{..........} =$ ....

Model tests on Geometry 2 nd preparatory

Model (1)

1 *Complete the following :*

The longest side in the right angled triangle is ....

If the lengths of two sides in a triangle are 2 cm. and 7 cm. then .... ∠the length of the third side < ....

If the measures of two angles in a triangle are different, then the greater in measure of them is opposite to ....

If the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length then ....

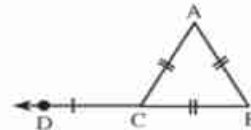
If the measure of an angle in the isosceles triangle equals 60° then the triangle is ....

2 *Choose the correct answer from those given :*

In the opposite figure :

Δ ABC is equilateral , then m (∠ ACD) = ...

(a) 45° (b) 60°

(c) 120° (d) 135°

In Δ ABC which is right angled at B , if AC = 20 cm. then the length of the median of the triangle drawn from B equals ....

(a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

XYZ is a triangle in which m (∠ Z) = 70° and m (∠ Y) = 60° then YZ .... XY

(a) > (b) < (c) = (d) twice

The lengths which can be lengths of sides of a triangle are

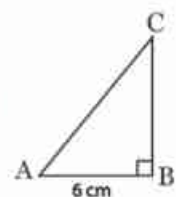
(a) (0 , 3 , 5) (b) (3 , 3 , 5) (c) (3 , 3 , 6) (d) (3 , 3 , 7)

The triangle in which the measure of two angles of it are 42° and 69° is ....

(a) an isoscles triangle. (b) an equilateral triangle.

(c) a scalene triangle. (d) a right angled triangle.

In the opposite figure, m (∠ A) = 2m (∠ C) , AC= .... cm.

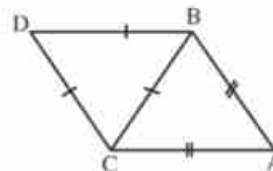
(a) 3 (b) 6 (c) 9 (d) 12

3 *Complete :*

[a] If Δ ABC in which AB > AC then m (∠ C) .... m (∠ B)

[b] In the opposite figure :

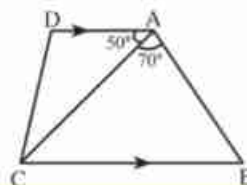
m (∠ A) = 50° , AB = AC

and Δ DBC is an triangle equilateral

Find m (∠ ABD)

[c] In the opposite figure :

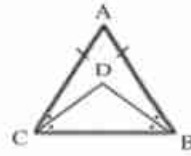
$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 70°

and m (∠ DAC) = 50°

Prove that : BC > AC

4 *[a]* Prove that the base angles of the isosceles triangle are congruent.

[b] In the opposite figure :

$AB = AC$, $\overrightarrow{BD}$ bisects $\angle B$ and $\overrightarrow{CD}$ bisects $\angle C$

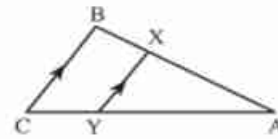
Prove that : Δ DBC is isosceles

5 *[a] In the opposite figure :*

Arrange the angles of Δ ABC descendingly due to their measures

[b] In the opposite figure :

$AB > BC$, $\overline{XY} \parallel \overline{BC}$

Prove that :

$AX > XY$

Model (2)

1 *Choose the correct answer from those given :*

1. The triangle which has three axes of symmetry is ....
   (a) scalene (b) isosceles (c) right angled (d) equilateral.
2. The sum of lengths of two sides in a triangle is .... the length of the third side.
   (a) greater than (b) smaller than (c) equals to (d) twice
3. If the lengths of two sides in an isosceles triangle are 8 cm. and 4 cm. then the length of the third side is ....
   (a) 4 (b) 8 (c) 3 (d) 12
4. In Δ ABC if $m(\angle B) = 130°$ then the tallest side of it is ....
   (a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median
5. Δ XYZ is an isosceles triangle in which $m(\angle X) = 100°$ then $m(\angle Y) =$ ....°
   (a) 100 (b) 80 (c) 60 (d) 40
6. In the opposite figure $x+y =$ ....°
   (a) 100 (b) 140 (c) 180 (d) 280

2 *Complete the following :*

1. If the measure of an angle in a right angled triangle is 45° then the triangle is ....
2. The length of any side in a triangle .... the sum of lengths of the two other sides.
3. If $\overline{AB} \equiv \overline{XY}$ then AB = ...
4. In Δ ABC of $m(\angle A) = 30°$ and $m(\angle B) = 90°$ then BC = ... AC
5. The axis of symmetry of a line segment is the straight line which .... at its midpoint.

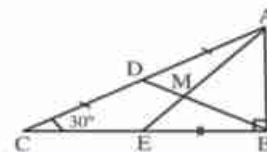
3 *[a]* In Δ ABC in which AB = 7 cm. , BC = 5 cm. and AC = 6 cm. Arrange its angles ascendingly due to their measure.

[b] In the opposite figure :

Δ ABC is right angled at B ,

$m(\angle C) = 30°$, D is the midpoint of $\overline{AC}$

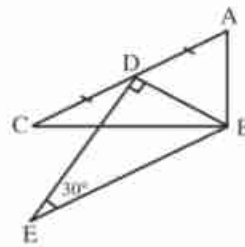
, E is the midpoint of $\overline{BC}$, AC = 9 cm.

Find the length of each of $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$

4 *[a] In the opposite figure :*

m (∠ ABC) = m (∠ BDE) = 90°

, m (∠ E) = 30°

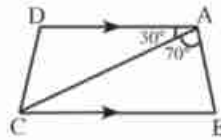
D is the midpoint of $\overline{AC}$

Prove that : AC = BE

[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 70°

, m (∠ DAC) = 30°

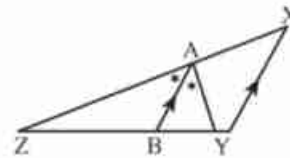
Prove that :

AC > BC

5 *[a]* If the measures of two angles of a triangle are different , then their greater in measure is opposite to ....

[b] In the opposite figure :

$\overline{AB}$ // $\overline{XY}$ and $\overrightarrow{AB}$ bisects ∠ YAZ

Prove that :

XZ > YZ

Model Test

" Merge students "

Questions (1) **Complete:**

1) The point of concurrence of the medians of the triangles divides each median in thr ratio of .... : .... from its ....

2) In the right-angled triangle, the length of the median drawn from of the right angle equals ....

3) The base angles of the isosceles triangle are ....

4) In Δ ABC, m (∠B) = 70°, m (∠C) = 50°, then AC .... AB

5) The median of an isosceles triangle from the vertex angle ....

Questions (2): **Choose the correct answer:**

1) If Δ ABC is an equilateral triangle, then m (∠B) = ... °

(a) 30° (b) 60° (c) 70° (d) 90°

2) The length of the side opposite to angle of 30°in the right-angled triangle = ... Hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 2

3) If the measure of the vertex angle of the isosceles triangle equals 80°, then the measure of one of the base angles = ... °.

(a) 60 (b) 40 (c) 30 (d) 50

4) The number of axes of symmetry of the isosceles triangle = ...

(a) 1 (b) 2 (c) 3 (d) 0

5) Δ ABC, m ($\angle$A) = 50°, m ($\angle$B) = 60°, then the longest side is ....

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$

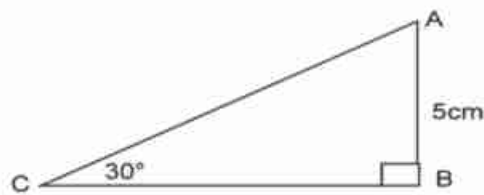
Questions (3): **In the opposite figure:**

ABC is right-angled triangle at B,

m ($\angle$C) = 30°, AB = 5 cm.

Find the length of $\overline{AC}$

$\because$ m ($\angle$ B) = ... ° , m ($\angle$ C) = ... °

$\therefore$ AB = $\frac{1}{2}$ x ....

$\therefore$ AC = ... cm

Questions (4):

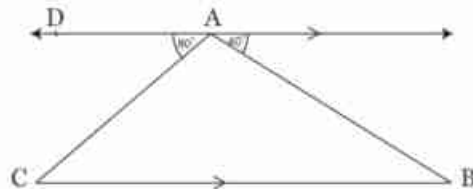
A) ΔABC, m ($\angle$ A) = 40°, m ($\angle$ B)= 75°, m ($\angle$ C) = 65°, arrange the lengths of the sides of this triangle descendingly the order is : ....

B) In the opposite figure:

$\overleftrightarrow{AD} // \overline{BC}$

Complete:

1- m($\angle$ B) = ... °

2- The longest side of ΔABC is ....

Questions (5): **In the opposite figure Put (✓) infront of the correct answer, (X) for the incorrect answer:**

AB = AC = CD = AD = 10 cm , m ($\angle$ BAC) = 70°

1) m ($\angle$ B) = 55° ( )

2) m ($\angle$ D) = 70° ( )

3) m ($\angle$ DCB) = 120° ( )

4) AB + AD = 20 cm ( )

5) AB + BC = BC + CD ( )